In this work, we present a comparable formula for the differential-delay equation (DDE):

$$\frac{dx}{dt} = \alpha x + \beta x_d + a_1 x^2 + a_2 x x_d + a_3 x_d^2 + b_1 x^3 + b_2 x^2 x_d + b_3 x x_d^2 + b_4 x_d^3 \tag{6}$$

where $x = x(t)$ and $x_d = x(t - T)$. Here T is the delay. Associated with (6) is a linear DDE

$$\frac{dx}{dt} = \alpha x + \beta x_d \tag{7}$$

We assume that (7) has a critical delay T_{cr} for which it exhibits a pair of pure imaginary eigenvalues $\pm\omega i$ corresponding to the solution

$$x = c_1 \cos \omega t + c_2 \sin \omega t \tag{8}$$

Then for values of delay T which lie close to T_{cr},

$$T = T_{cr} + \mu \tag{9}$$

the nonlinear Eq. (6) may exhibit a periodic solution which can be written in the approximate form:

$$x = A \cos \omega t \tag{10}$$

where the amplitude A can be obtained from the following expression for A^2:

$$A^2 = \frac{P}{Q}\mu \tag{11}$$

where

$$P = 4\beta^3(\beta - \alpha)(\beta + \alpha)^2(-5\beta + 4\alpha) \tag{12}$$

$$\begin{aligned} Q = {} & 15b_4\beta^6 T_{cr} + 5b_2\beta^6 T_{cr} + 3\alpha b_4\beta^5 T_{cr} - 15\alpha b_3\beta^5 T_{cr} + \alpha b_2\beta^5 T_{cr} - 15\alpha b_1\beta^5 T_{cr} - 22a_3^2\beta^5 T_{cr} \\ & - 7a_2a_3\beta^5 T_{cr} - 14a_1a_3\beta^5 T_{cr} - 3a_2^2\beta^5 T_{cr} - 7a_1a_2\beta^5 T_{cr} - 4a_1^2\beta^5 T_{cr} - 12\alpha^2 b_4\beta^4 T_{cr} - 3\alpha^2 b_3\beta^4 T_{cr} \\ & + 6\alpha^2 b_2\beta^4 T_{cr} - 3\alpha^2 b_1\beta^4 T_{cr} + 12a_3^2\alpha\beta^4 T_{cr} + 37a_2a_3\alpha\beta^4 T_{cr} + 30a_1a_3\alpha\beta^4 T_{cr} + 7a_2^2\alpha\beta^4 T_{cr} \\ & + 19a_1a_2\alpha\beta^4 T_{cr} + 18a_1^2\alpha\beta^4 T_{cr} + 12\alpha^3 b_3\beta^3 T_{cr} + 2\alpha^3 b_2\beta^3 T_{cr} + 12\alpha^3 b_1\beta^3 T_{cr} + 4a_3^2\alpha^2\beta^3 T_{cr} \\ & - 20a_2a_3\alpha^2\beta^3 T_{cr} - 16a_1a_3\alpha^2\beta^3 T_{cr} - 12a_2^2\alpha^2\beta^3 T_{cr} - 26a_1a_2\alpha^2\beta^3 T_{cr} - 8a_1^2\alpha^2\beta^3 T_{cr} - 8\alpha^4 b_2\beta^2 T_{cr} \\ & - 4a_2a_3\alpha^3\beta^2 T_{cr} + 8a_2^2\alpha^3\beta^2 T_{cr} + 8a_1a_2\alpha^3\beta^2 T_{cr} + 5b_3\beta^5 + 15b_1\beta^5 - 15\alpha b_4\beta^4 + \alpha b_3\beta^4 - 15\alpha b_2\beta^4 \\ & + 3\alpha b_1\beta^4 - 4a_3^2\beta^4 - 9a_2a_3\beta^4 - 18a_1a_3\beta^4 - a_2^2\beta^4 - 9a_1a_2\beta^4 - 18a_1^2\beta^4 - 3\alpha^2 b_4\beta^3 + 6\alpha^2 b_3\beta^3 \\ & - 3\alpha^2 b_2\beta^3 - 12\alpha^2 b_1\beta^3 + 26a_3^2\alpha\beta^3 + 19a_2a_3\alpha\beta^3 + 30a_1a_3\alpha\beta^3 + 11a_2^2\alpha\beta^3 + 33a_1a_2\alpha\beta^3 + 12a_1^2\alpha\beta^3 \\ & + 12\alpha^3 b_4\beta^2 + 2\alpha^3 b_3\beta^2 + 12\alpha^3 b_2\beta^2 - 8a_3^2\alpha^2\beta^2 - 32a_2a_3\alpha^2\beta^2 - 12a_1a_3\alpha^2\beta^2 - 14a_2^2\alpha^2\beta^2 \\ & - 18a_1a_2\alpha^2\beta^2 - 8\alpha^4 b_3\beta - 8a_3^2\alpha^3\beta + 8a_2a_3\alpha^3\beta + 4a_2^2\alpha^3\beta + 8a_2a_3\alpha^4 \end{aligned} \tag{13}$$

In Eq. (11), A is real so that $A^2 > 0$, which means that μ must have the same sign as $\frac{P}{Q}$.

Eq. (13) depends on μ, α, β, a_i, b_i and T_{cr}. This equation may be alternately written with T_{cr} expressed as a