Math 2930 Lecture #1

Outline of Course

Definitions: The order of a D.E. (differential equation) is the order of the highest derivative which it contains.

ODE = ordinary D.E.
PDE = partial D.E.

Examples

(1) $$\frac{dy}{dx} = y + x + y^2$$ is 1^{st} order ODE

(2) $$\frac{d^2x}{dt^2} + x = \sin t$$ is 2^{nd} order ODE

(3) $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + u$$ is 2^{nd} order PDE

In example (1), the term $\frac{dy}{dx}$ involves

y as a function of x

y ↑ the dependent variable

x ↑ the independent variable

[because y depends on x]

In example (3), u is the dependent variable
x, t are independent variables

D.E.'s are either LINEAR or NONLINEAR.

How can you tell the difference?

It depends on how the dependent variable appears in the D.E.

The D.E. is linear if it is the sum of terms, each of which is of the form

[here we take y as dependent variable
and x as independent variable]

$$f(x)\,\frac{d^n y}{dx^n} \quad \text{or just } f(x)$$

No y here

including the special cases in which

$f(x)$ is a constant (so its not actually a function of x)

and in which $n = 0$

(that is, the zeroth derivative = y itself)

The most general 1st order linear ODE is

$$f_1(x)\frac{dy}{dx} + f_2(x)\,y + f_3(x) = 0$$

where $f_i(x)$ is a given fn.

Example (1) is NONLINEAR because of the y^2 term.

Examples (2), (3) are LINEAR.

Using these definitions, we can outline the course as follows:

Part I: 1^{st} order ODE's (both linear and nonlinear)
(Example 1)

Part II: 2^{nd} order ODE's (linear only)
(Example 2)

Part III: 2^{nd} order PDE's (linear only)
(Example 3)

What does it mean to SOLVE a D.E.?

Example

$$\frac{dy}{dx} = \sin x$$

We are to find a function $y(x)$ which "SATISFIES" the D.E.
That is, when the function $y = f(x)$ is substituted into the D.E., we get an identity.

To solve this DE, integrate both sides with respect to (= w.r.t.) x:

$$\int \frac{dy}{dx}\, dx = \int \sin x \, dx$$

$$y = -\cos x + C$$

an arbitrary constant, coming from the indefinite integral

Language of DE's:
We have "integrated" the DE and have obtained its "solution".

Q: How do we find the constant of integration C?

A: We need an extra equation to find C.

It accompanies the D.E. and is called an

initial condition (I.C.)

or a boundary condition (B.C.)

Example

$$\frac{dy}{dx} = \sin x \leftarrow \text{the D.E.}$$

$$y = 0 \text{ when } x = 0 \leftarrow \text{the I.C.}$$

So far we have

$$y = -\cos x + C$$

(called the "general" solution)

To find C we use the I.C.

$$y = \underset{\substack{\uparrow \\ \text{set } y=0}}{0} = -\cos \underset{\substack{\uparrow \\ \text{set } x=0}}{0} + C = -1 + C$$

Then solve for C: $0 = -1 + C \Rightarrow C = 1$

$\therefore$ The solution to the D.E. + I.C. is

$$y = -\cos x + 1$$