Constant Coefficient Linear ODE's

We will be interested in solving ODE's of the form:

$$a \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + c y = f(t)$$

where a, b, c are given constants.

The solution can always be broken up into two parts:

$$y = y_h + y_p$$

where h stands for "homogeneous" and
p " " "particular"

where y_h is the general solution of the "homogeneous equation":

$$a \frac{d^2 y_h}{dt^2} + b \frac{dy_h}{dt} + c y_h = 0$$

and where y_p is a particular solution of the "nonhomogeneous equation":

$$a \frac{d^2 y_p}{dt^2} + b \frac{dy_p}{dt} + c y_p = f(t)$$

Example p. 141, problem 1

$$\frac{d^2 y}{dt^2} - 2\frac{dy}{dt} - 3y = 3e^{2t}$$

We write $\quad y = y_h + y_p$

First work on y_h which satisfies the homogeneous eq:

$$\frac{d^2 y_h}{dt^2} - 2\frac{dy_h}{dt} - 3y_h = 0$$

Look for a solution in the form $y_h = e^{rt}$ where r is to be found

$$r^2 e^{rt} - 2re^{rt} - 3e^{rt} = 0$$

$$\underbrace{(r^2 - 2r - 3)}_{=0} e^{rt} = 0$$

$$\Rightarrow (r-3)(r+1) = 0 \Rightarrow r = 3, -1$$

We have found 2 solutions: $y_h = e^{3t}$ and $y_h = e^{-t}$.

The general solution will be a "linear combination" of the two solutions:

$$y_h = c_1 e^{3t} + c_2 e^{-t}$$

where c_1 and c_2 are arbitrary constants.

The particular solution y_p must satisfy

$$\frac{d^2y_p}{dt^2} - 2\frac{dy_p}{dt} - 3y_p = 3e^{2t}$$

We look for a solution in the form

$$y_p = Ae^{2t}$$

where A is to be found by substituting into

We obtain

$$[4A - 2(2A) - 3A]e^{2t} = 3e^{2t}$$

or

$$-3A = 3 \Rightarrow A = -1$$

so that

$$y_p = -e^{2t}$$

<u>Summary</u> The general solution to

$$y'' - 2y' - 3y = 3e^{2t}$$

is

$$y = y_h + y_p$$
$$= c_1e^{3t} + c_2e^{-t} - e^{2t}$$