There is doubtless a layer of less strongly excited gas in the neighbourhood of the walls of any vacuum tube; but the fact that the observation of reversal seems to depend on a high current density appears to imply that the existence of this layer is not sufficient to explain the phenomena.

July 31st, 1926.

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# LXXXVIII. *On* "*Relaxation-Oscillations.*"

*By* BALTH. VAN DER POL, Jun., *D.Sc.**

1. THE condition of a simple oscillatory system, possessing one degree of freedom and subjected to a dissipative force, may be represented by the well-known linear differential equation

$$\ddot{x}+\alpha\dot{x}+\omega^2x=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the solution of which is

$$x=C_1e^{-\frac{\alpha t}{2}}\sin\left(\sqrt{\omega^2-\frac{\alpha^2}{4}}\,t+\phi\right). \quad . \quad . \quad . \quad (2)$$

If we have also $\alpha>0$

and $\frac{\alpha^2}{4}<\omega^2$,

the solution (2) represents a damped oscillation with a logarithmic decrement $\delta$ given by

$$\frac{\delta}{\pi}=\frac{\alpha}{\omega}.$$

If it happens that the "resistance" in the system is negative, such as may be the case in certain electrical circuits, the sign of $\alpha$ is reversed and (1) becomes

$$\ddot{x}-\alpha\dot{x}+\omega^2x=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the solution of which is

$$x=C_1e^{+\frac{\alpha t}{2}}\sin\left(\sqrt{\omega^2-\frac{\alpha^2}{4}}\,t+\phi\right).$$

If, again, we have $\alpha>0$

and $\frac{\alpha^2}{4}<\omega^2$,

the solution (2 *a*) represents an oscillation; but in this case the amplitude is gradually increasing instead of decreasing,

* Communicated by the Author.



and the logarithmic decrement of the former case is replaced by a logarithmic increment $\delta$ given by

$$\frac{\delta}{\pi}=\frac{\alpha}{\omega}.$$

2. A solution of the form (2 *a*) is, however, physically unrealizable because it indicates an amplitude growing to infinity. Thus for actual physical systems the differential equation (3) will only be valid for values of $x$ up to a certain value. To express the limitation of the amplitude we must assume that the coefficient of the "resistance" term is a function of the amplitude itself, becoming positive at the higher values. Thus we may in (3) replace $\alpha$ by the expression $\alpha-3\gamma x^2$, where $\gamma$ is a constant. Hence we obtain instead of (3):

$$\ddot{x}-(\alpha-3\gamma x^2)\dot{x}+\omega^2x=0. \quad . \quad . \quad . \quad . \quad (4)$$

This equation has been previously considered* in connexion with the subject of triode oscillations.

Let us now change the units of time and of $x$ and write

$$\left.\begin{aligned}\omega t&=t',\\ \text{and}\qquad x&=\sqrt{\frac{\alpha}{3\gamma}}\,v.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4\,a)$$

Then (4) becomes (after dropping the accents)

$$\frac{d^2v}{dt^2}-\frac{\alpha}{\omega}(1-v^2)\frac{dv}{dt}+v=0. \quad . \quad . \quad . \quad . \quad (5)$$

Writing further

$$\frac{\alpha}{\omega}=\epsilon$$

and using fluxional notation we have

$$\ddot{v}-\epsilon(1-v^2)\dot{v}+v=0. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now in the usual cases of triode oscillations we know from the experimental data that it takes several periods for the amplitude to build up to the final steady value. Expressed mathematically this means

$$\epsilon\ll1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

* Van der Pol, *Tijdsch. v. h. Ned. Radio Gen.* i. (1920); Radio Review, i. p. 701 (1920). Appleton and van der Pol, Phil. Mag. xliii. p. 177 (1922). Robb, Phil. Mag. xliii. p. 206 (1922).

Recognizing the condition (7) the equation (6) may be solved approximately in the following way:

Let $v = a \sin(t+\phi)$,

in which, since the logarithmic increment is small, we may assume $a$ to be a slowly varying function of the time, such that

$$\dot{a} \ll a.$$

On substituting (8) in (6), and omitting small quantities and neglecting higher harmonics, we find that

$$\frac{1}{\epsilon}\frac{da^2}{dt} - a^2 + \frac{1}{4}a^4 = 0,$$

or

$$a^2 = \frac{4}{1+e^{-\epsilon(t+C)}}.$$

The approximate solution of (6) may therefore be written

$$v = \frac{2\sin(t+\phi)}{\sqrt{1+e^{-\epsilon(t+C)}}},$$

which represents an oscillation the amplitude of which at first* increases with time according to the factor $e^{\epsilon\frac{t}{2}}$, but finally approaches the steady value

$$a = 2.$$

Reverting to the original variables of (4) we thus have

$$x = \sqrt{\frac{\alpha}{\frac{3}{4}\gamma}} \cdot \frac{1}{\sqrt{1+e^{-\alpha(t+C)}}} \cdot \sin(\omega t+\phi). \quad . \quad . \quad (8)$$

There is, however, a more direct method of finding the steady final amplitude of the oscillations for cases represented by (6). For example, let us assume that a periodic solution of the equation exists. Multiplying (6) throughout by $\int v\,dt$ and integrating over the (unknown) period we find that

$$\overline{v^2} = \tfrac{1}{3}\overline{v^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

(the horizontal dashes indicating integration over the period). If now we assume $v$ to be very nearly sinusoidal, *i. e.*

$$v = a \sin t,$$

(9) gives us at once

$$a = 2.$$

* *I. e.*, so long as the amplitude is so small that in (6) the non-linear term $\epsilon v^2 \dot{v}$ may be neglected in comparison with $-\epsilon\dot{v}$.



To find the time period of the oscillations we multiply (6) by $v$ and again integrate over the (unknown) period. In this case we find that

$$\overline{\dot{v}^2} = \overline{v^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

so that, assuming again the solution to be approximately sinusoidal, we find from (10), that the angular frequency is unity.

3. Up to the present we have considered (6) with the supplementary condition

$$\epsilon \ll 1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

but it is of considerable interest, and also the main object of this paper, to investigate the sequence of events when

$$\epsilon \gg 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\,a)$$

It may be noted that even with the new condition (7 *a*) the equation (6) has a periodic solution, since the relations (9) and (10) are independent of the numerical value of $\epsilon$. Further, on physical grounds, we may expect that there is a periodic solution of (6) when (7 *a*) holds, as may be seen from the following considerations. For small values of $v$, (6) may be written approximately as

$$\ddot{v} - \epsilon\dot{v} + v = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and this has, when (7 *a*) holds, an approximate solution

$$v = C_1 e^{\epsilon t} + C_2 e^{\frac{t}{\epsilon}}.$$

The value of $v$ therefore would approach infinity asymptotically so that $v=0$ is not a stable solution. Thus, so long as (11) is valid, we are dealing with the well-known *aperiodic* case, but with negative damping. But when the amplitude increases and $v^2 > 1$, the coefficient of the second term on the left-hand side of (6) becomes positive indicating a positive resistance and therefore a reduction in amplitude with time. Now the limiting values $v = \pm 1$ are not solutions of (6), so that, in general, we may expect the solutions to be periodic, *even when condition* (7 *a*) *is fulfilled*. Although for small amplitudes the resistance has such a big negative value that the linear case would be highly aperiodic, the non-linear term in (6), i. e. $v^2\dot{v}$, makes the solution periodic. We may thus say that we are dealing with a *quasi-aperiodic* solution.

4. It has not been found possible to obtain an approximate analytical solution for (6) with the supplementing

condition (7 *a*), but a graphical solution may easily be found in the following way. If we write

$$\dot{v}=z,$$

(6) may be written

$$\frac{dz}{dv}-\epsilon(1-v^2)+\frac{v}{z}=0, \quad . \quad . \quad . \quad . \quad (12)$$

which is a first order equation of the super-Riccatti type. Let us draw in a $z, v$ plane a series of "isoclynes," *i. e.* curves connecting all points for which $\frac{dz}{dv}$ is equal to a certain quantity. An example of such isoclynes is denoted by

$$\frac{dz}{dv}=C_1$$

where $C_1$ is a constant so that, combining with (12), we have as expression for an isoclyne:

$$C_1-\epsilon(1-v^2)+\frac{v}{z}=0.$$

Several of these isoclynes may be drawn in the $z, v$ plane, and we can indicate by means of short lines, as is done in fig. 1, the direction the integral curve must have when it crosses an isoclyne. (For example, this direction for the isoclyne $C_1$ is given by

$$\frac{dz}{dz}=C_1, \text{ etc.})$$

From a diagram in which the inclinations of the integral curves are marked on the isoclynes we may easily draw the integral curves in the $z, v$ plane. This is done in fig. 1 for the value

$$\epsilon=0{\cdot}1,$$

and it is seen that the integral curve obtained indicates the track of a point approaching a closed curve after many complete circuits (small increment).

In the same way fig. 2 is drawn for

$$\epsilon=1,$$

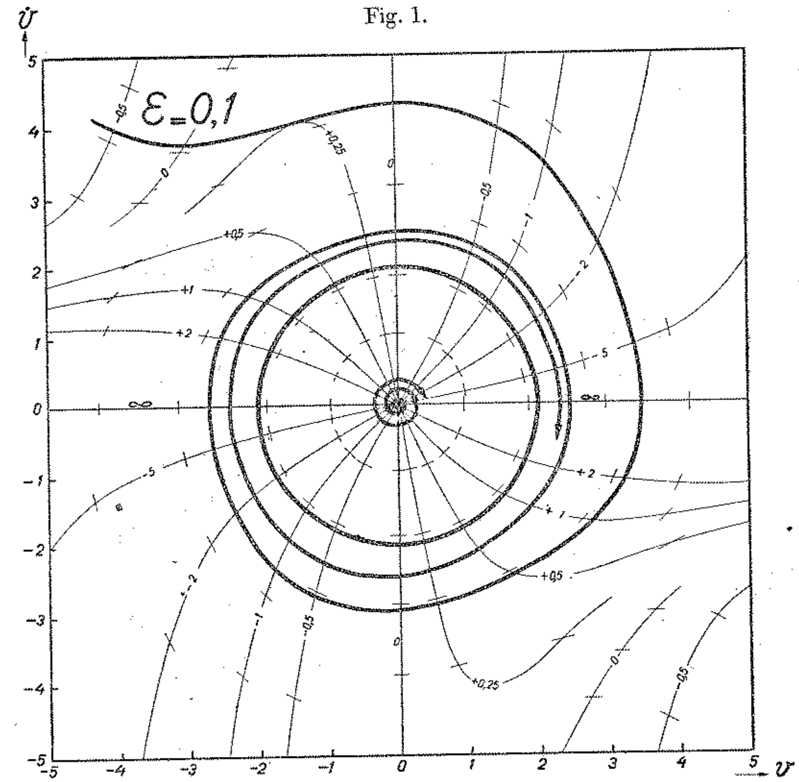
and fig. 3 for

$$\epsilon=10,$$

all three figures showing a closed curve solution. In the



Fig. 1.




case of fig. 3 (quasi-aperiodic case) the final steady closed curve, representing the periodic solution, is practically reached after one revolution only. When the intermediate integral $z=f(v)$ has been obtained in this way it is easy to construct from it in a similar fashion the integral

$$v=\phi(t).$$

Some examples of such results are exhibited in fig. 4 and represent the solution of (6) for the three cases

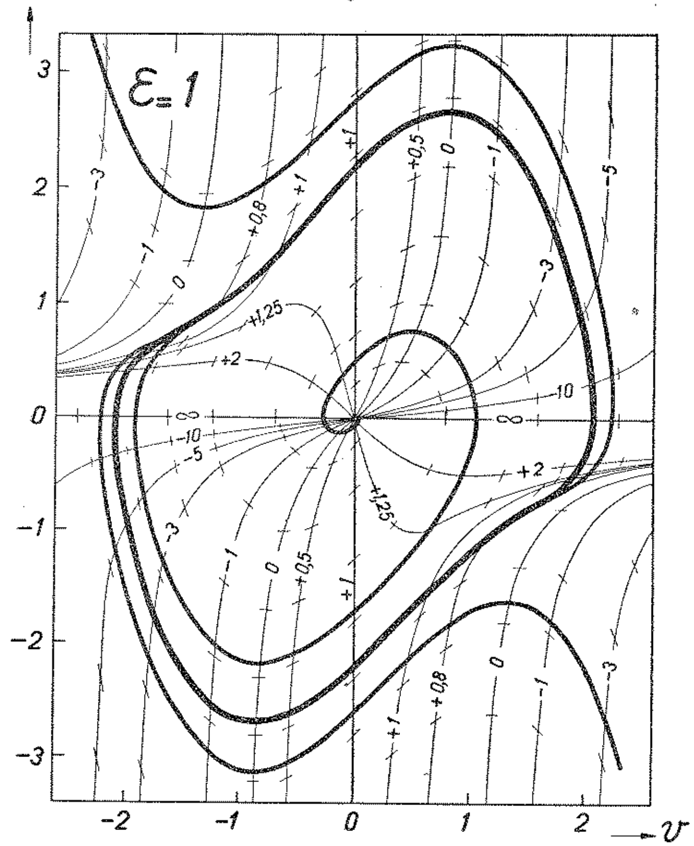
$$\epsilon=0{\cdot}1,$$

$$\epsilon=1,$$

$$\epsilon=10.$$

The first case (small increment) might also have been plotted from the solution (8), and a numerical comparison of the two solutions shows them to be in satisfactory agreement. This case represents a sinusoidal oscillation of gradually increasing amplitude, the value of which is finally

Fig. 2.




steady and equal to 2. The angular frequency, for the units used, is unity. The second case of fig. 4, (i. e. $\epsilon = 1$) indicates a somewhat similar sequence of events, but here the final amplitude is reached in fewer oscillations, while a marked departure from the sinusoidal form is noticed. The third case (i. e. $\epsilon = 10$) is particularly interesting. Here it is noticed that the curve first rises asymptotically and after



Fig.3 .

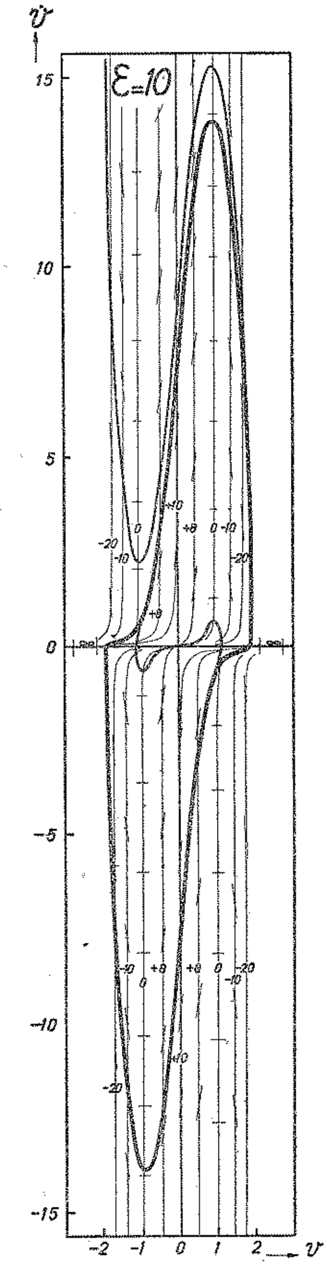
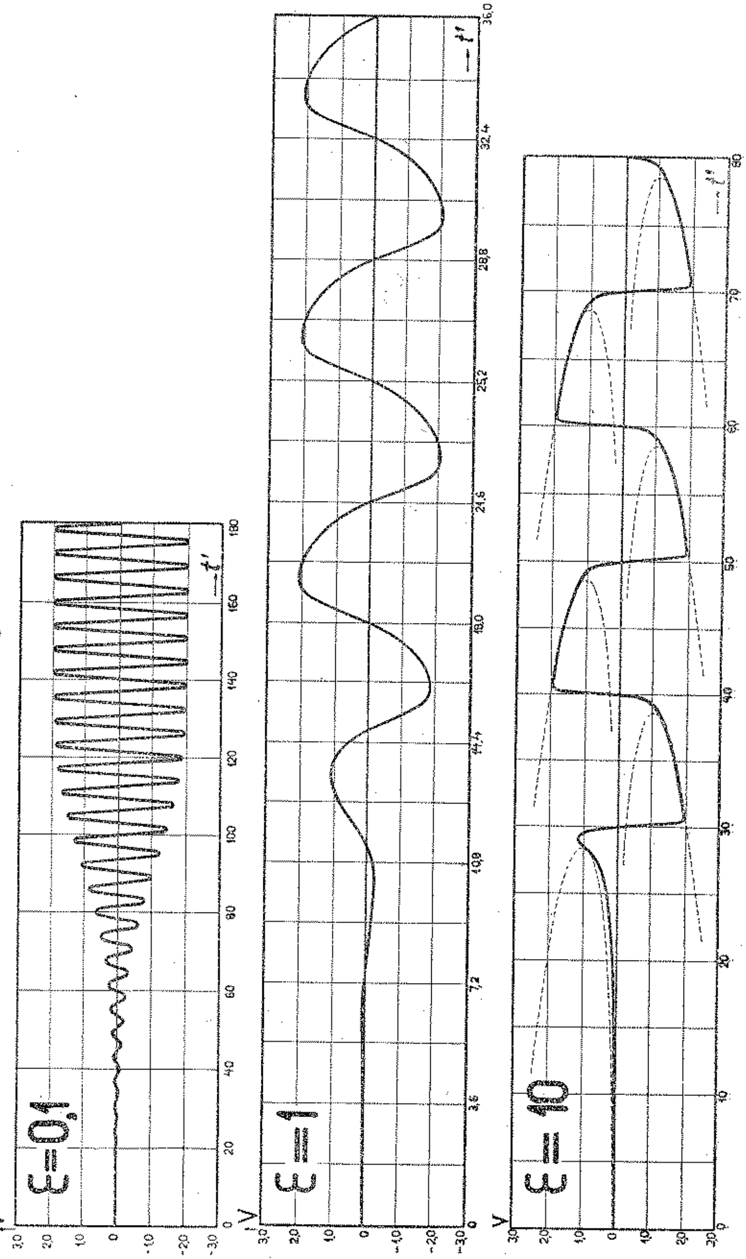


Fig. 4.






only one period practically reaches the final steady state. This steady state is characterized by a very marked departure from the sinusoidal form. It is seen that the amplitude alters very slowly from the value 2 to the value 1 and then very suddenly it drops to the value $-2$. Next we observe a very gradual increase from the value $-2$ to the value $-1$ and again a sudden jump to the value 2. This cycle term proceeds indefinitely.

Obviously this form of oscillation contains many higher harmonics of considerable amplitude. As will be seen later, the period T, instead of being $2\pi$ (as was the case when $\epsilon \ll 1$) increases with increase of $\epsilon$, and when $\epsilon \gg 1$ becomes equal to approximately $\epsilon$ itself; that is, we have

$$T \doteq \epsilon.$$

Let us now consider more closely the physical factors determining the value of the period T. If our equation (1) represents the circulation of electricity in a system of resistance R, capacity C, and inductance L, we have, as usual,

$$\alpha = \frac{R}{L},$$

$$\omega^2 = \frac{1}{LC}.$$

Now for the time period expressed in units of $t'$ we have already

$$T \doteq \epsilon,$$

so that, expressed in terms of $t$, the period becomes

$$T = \frac{\alpha}{\omega^2} = RC, \quad \ldots \ldots \quad (13)$$

which is a *time of relaxation* (time constant).

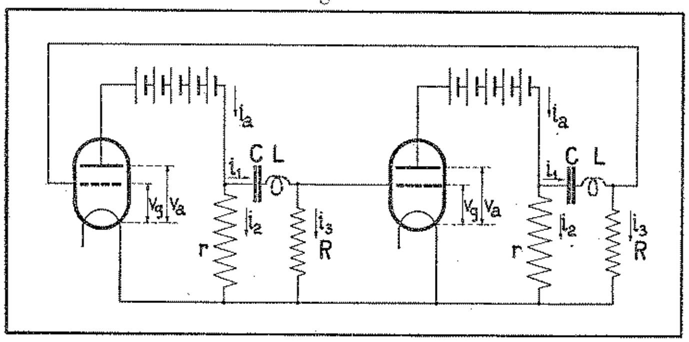
Thus our equation (6) for the quasi-aperiodic case, which differs considerably from the normal approximately sinusoidal solution, has again a purely periodic solution, the time period of which is expressed by the time of relaxation of the system. For this reason the term *relaxation-oscillation* is suggested for this phenomenon.

5. A type of oscillation previously described by Abraham & Bloch* is an example of relaxation-oscillations. These authors used an electrical system comprising two triodes and resistances and capacities only, which system

* Abraham & Bloch, *Ann. de Physique*, xii. p. 237 (1919).

they called "multivibrateur" because of the many higher harmonics of appreciable amplitude which it produced when oscillating. This system, which may be regarded as a two-stage resistance-capacity coupled amplifier with the output coupled back to the input side, is shown in a slightly modified form in fig. 5. In their original description of the system Abraham & Bloch draw attention to the fact that the time period of the oscillations produced by the multi-vibrator is approximately equal to the product RC (see fig. 5), but, so far as I am aware, no theoretical discussion of the way in which the oscillations are *maintained* has been published.

Fig. 5.




From the symmetrical value of the circuit we may expect that the two triodes can vibrate in exactly opposite phases. It is further known that the potentials and anode currents experience temporal variations closely represented by fig. 4 ($\epsilon=10$). Now, in order to explain the reason for the maintenance of oscillation in this system containing resistances and capacities only, we found it necessary to take into account the inductance L of the wires connected to the two capacities. (These are represented by the dotted lines L in fig. 5.) With the notation of the latter figure the current and potential departures from the unstable equilibrium values are given by the following equation :

$$\left.\begin{aligned} -v_{a1}=r_1 i_1 &= \left(\mathrm{R}+\mathrm{L}\frac{d}{dt}+\frac{1}{\mathrm{C}}\int dt\right)i_3 \\ \mathrm{R}i_3 &= -v_{g2} \\ i_{a1}=i_1+i_3 &= \phi(v_{g1}), \end{aligned}\right\}, \quad . \quad . \quad (14)$$

where $\phi(v_{g1})$ denotes the characteristic of the first triode round the equilibrium position and where, for simplicity,



the influence of the anode potential on the anode current is neglected. Further, assuming that the triodes of this symmetrical system are exactly equal and vibrate in opposite phase, we have

$$v_{g1}=-v_{g2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

From (14) and (15) we derive

$$\left(\mathrm{L}\frac{d}{dt}+(\mathrm{R}+r)+\frac{1}{\mathrm{C}}\int dt\right)v_{g1}=\mathrm{R}r\phi(v_{g1}). \quad . \quad (16)$$

Now, as an approximate expression for the characteristic

$$i_{a1}=\phi(v_{g1}),$$

we may take again the third order parabola

$$i_{a1}=\mathrm{S}\left(1-\frac{v_{g1}^2}{v_{g0}^2}\right)v_{g1}, \quad . \quad . \quad . \quad . \quad (17)$$

where S is the slope of the anode-current/grid-voltage curve.

Usually the anode lead resistance $r$ is small compared with the grid leak R, *i. e.*

$$r \ll \mathrm{R},$$

so that from (16) and (17) we have

$$\mathrm{L}\ddot{v}_{g1}-\mathrm{R}\left\{(r\mathrm{S}-1)-r\mathrm{S}\frac{v_{g1}^2}{v_{g0}^2}\right\}\dot{v}_{g1}+\frac{1}{\mathrm{C}}v_{g1}=0. \quad (18)$$

From (18) we note that the "resistance" of the system for small amplitudes is only negative so long as

$$r\mathrm{S}>1,$$

so that we have here the approximate condition for the production of oscillations. If we further make the following substitution :

$$\left.\begin{aligned} t &= t'\sqrt{\mathrm{CL}} \\ \frac{v_{g1}}{v_{g0}} &= v\sqrt{\frac{r\mathrm{S}-1}{r\mathrm{S}}} \\ \epsilon &= \mathrm{R}(r\mathrm{S}-1)\sqrt{\frac{\mathrm{C}}{\mathrm{L}}} \end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad (19)$$

(18) may be written

$$\ddot{v}-\epsilon(1-v^2)\dot{v}+v=0, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is the equation originally discussed and which may be said to be the representative differential equation of the multivibrator ($\epsilon \gg 1$). But we may note that, in order to represent the action of the multivibrator by this equation, we had to take into account the small inductance L of the wires connected to the capacities C, as shown in fig. 5.

We further note from (19) that $\epsilon$ depends on the value of $L^{-\frac{1}{2}}$ so that $\epsilon$ increases without limit the further L is decreased. We may make a rough estimate of $\epsilon$ for the following practical values :

$$R=10^5 \text{ ohms}, \ rS-1=3, \ C=0{\cdot}01 \ \mu\text{fd}, \ L=10 \text{ cm}.$$

In this case,

$$\epsilon=10^5 \,.\, 3 \,.\, \sqrt{\frac{10^{-8}}{10^{-8}}}=3 \,.\, 10^5,$$

so that the conduction

$$\epsilon \gg 1$$

is certainly satisfied. The smaller the value of the residual inductance L, the greater is this inequality.

*It therefore may be concluded that the special vibration of the multivibrator represents an example of a general type of relaxation-oscillations and that in order to explain the maintenance of the oscillations we have to take into account the residual inductance of the system* (as shown in fig. 5), *however small this may be.*

The result of the above discussion illustrates how the form of the solution of differential equations like the representative equal (18) or (6) is entirely altered by taking into account a term with an infinitesimal coefficient (*e. g.* the first term $L\ddot{v}_{g1}$ in (18)).

But the physical reason for taking this into account becomes immediately apparent when we solve (18) or (6) omitting the first term. In this case the solution of (6) is

$$\log v^2 - v^2 = \frac{2t+C}{\epsilon}, \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which is represented graphically in fig. 6. We note that the value of $v^2$ first increases exponentially with time, but near $v^2=1$ the curve bends upwards with an infinite slope, there being further no solution in the real domain. When the inductance is thus disregarded both $\dot{v}$ and $\ddot{v}$ become infinite when $v=\pm 1$, so that the condensers C (fig. 5) would acquire a very large charge in an indefinitely small time. But these infinitely sudden changes of current are prevented by the presence of the small residual inductance.

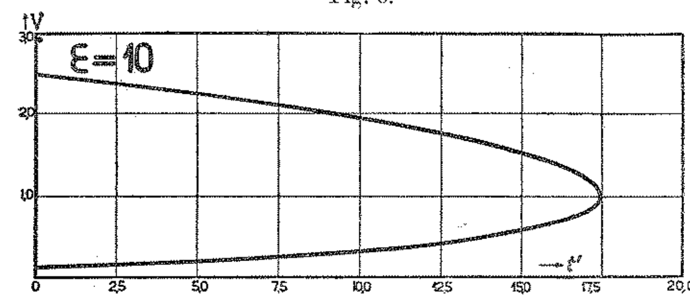
This is illustrated by the fact that the solution (20) for the derivation of which the inductance has been neglected, and which is represented by the repeated dotted lines in



fig. 4 ($\epsilon=10$), is found to coincide approximately with the exact solution so long as the slope of $v$ is small. Whenever, however, the slope tends to become infinite we note that the term $\ddot{v}$ in (6) becomes of importance and keeping the slope finite maintains the multivibrator in vibration.

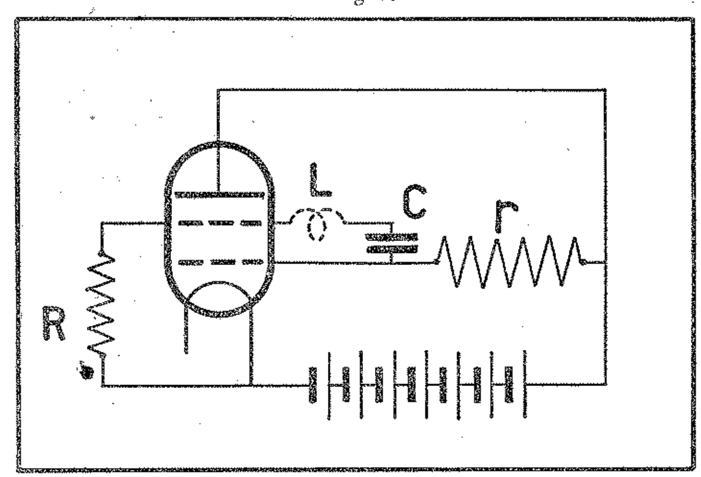
The multivibrator may therefore be compared to a double steam engine with a flywheel which is much too small. The small inertia of the flywheel must, however, be present to carry the system beyond its equilibrium (dead) points.

Fig. 6.




§ 6. Now that the general equation (4) for *relaxation-oscillations* is known, it is an easy matter to devise further electrical systems of the same type. One such a system is

Fig. 7.




depicted in fig. 7 and comprises a tetrode, two resistances $r$ and R, and a condenser C. If we again recognize the inductance L of the wire connecting the condenser to the outer grid, we find that the circulation of electricity in the system is represented by (6). An experimental test of the system showed that the system produced oscillations with a time period approximately equal to CR. When $r$ is replaced by a telephone receiver the oscillations become audible and the change of frequency by changing R or C is easily demonstrated.

7. Finally it seems quite likely that, when the total characteristic (including the parts with a negative slope) is taken into account, the well-known vibration of a neon-tube connected to a resistance and condenser in shunt * may be similarly treated under the heading of relaxation-oscillations.

Similarly, (though no detailed investigation has been carried out) it is likely that the oscillations of a "Wehnelt" interrupter belong to the general class of relaxation-oscillations and perhaps also heart-beats.

Eindhoven, 6th May, 1926.
Physical Laboratory,
Philips' Glowlampworks, Ltd.

* See, *e. g.* Schallreuter, 'Ueber Schwingungserscheinungen in Entladungsröhren.' Braunschweig, 1923.

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## LXXXIX. *The Crystalline Structure of Anhydrite.*

*By* Prof. JARL A. WASASTJERNA †.

IN a particularly interesting paper published in the July number of this Journal, Messrs. Dickson and Binks examined the structure of Anhydrite. In a note added, it is stated that the present author had previously examined the same problem (*Societas Scientiarum Fennica : Commentationes Physico-Mathematicæ*, ii. p. 26, 1925), and that the two investigations produced structures of the same type but of somewhat different parameter values. From that the conclusion might be drawn, that the two pieces of work stand in certain opposition to each other, if only in detail. But that can hardly be said to be the case.

By virtue of a method of elimination, carried through consistently, which takes every type of structure, mathematically possible, into consideration (in this case there are 85 such different types), the present author has shown that in respect of anhydrite there are two, and only two, possible structures. In the present author's work these structures are denoted as IV and V (pp. 39–40). Let us call them A and B. These two structures are of the same type and extremely closely related to one another. A displacement of the atom groups in the direction of the $a$-axis of 0·05 transforms one structure to the other.

† Communicated by Prof. W. L. Bragg, F.R.S.



The atomic coordinates are :—

| | | |
|---|---|---|
| Ca : | $[[q\cdot\frac{1}{4}\cdot 0]]$ | $[[\bar{q}\cdot\frac{3}{4}\cdot 0]]$ |
| | $[[q+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}]]$ | $[[\bar{q}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2}]]$ |
| S : | $[[r\cdot\frac{1}{4}\cdot 0]]$ | $[[\bar{r}\cdot\frac{3}{4}\cdot 0]]$ |
| | $[[r+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}]]$ | $[[\bar{r}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2}]]$ |
| O : | $[[f\cdot g\cdot 0]]$ | $[[\bar{f}\cdot g+\frac{1}{2}\cdot 0]]$ |
| | $[[f\cdot\bar{g}+\frac{1}{2}\cdot 0]]$ | $[[\bar{f}\cdot\bar{g}\cdot 0]]$ |
| | $[[f+\frac{1}{2}\cdot g\cdot\frac{1}{2}]]$ | $[[\bar{f}+\frac{1}{2}\cdot g+\frac{1}{2}\cdot\frac{1}{2}]]$ |
| | $[[f+\frac{1}{2}\cdot\bar{g}+\frac{1}{2}\cdot\frac{1}{2}]]$ | $[[\bar{f}+\frac{1}{2}\cdot\bar{g}\cdot\frac{1}{2}]]$ |
| | $[[h\cdot\frac{1}{4}\cdot k]]$ | $[[\bar{h}\cdot\frac{3}{4}\cdot\bar{k}]]$ |
| | $[[h\cdot\frac{1}{4}\cdot\bar{k}]]$ | $[[\bar{h}\cdot\frac{3}{4}\cdot k]]$ |
| | $[[h+\frac{1}{2}\cdot\frac{1}{4}\cdot k+\frac{1}{2}]]$ | $[[\bar{h}+\frac{1}{2}\cdot\frac{3}{4}\cdot\bar{k}+\frac{1}{2}]]$ |
| | $[[h+\frac{1}{2}\cdot\frac{1}{4}\cdot\bar{k}+\frac{1}{2}]]$ | $[[\bar{h}+\frac{1}{2}\cdot\frac{3}{4}\cdot k+\frac{1}{2}]]$ |

We introduce the following new parameters :—

$$f=r+t_1$$
$$g=\tfrac{1}{4}-u_1$$
$$h=r-t_2$$
$$k=u_2.$$

The parameters acquire the following values according to Messrs. Dickson and Binks and to the present author :—

| Wasastjerna. | Dickson and Binks *. |
|---|---|
| $t_1=0·16$ | $t_1=0·15$ |
| $t_2=0·16$ | $t_2=0·15$ |
| $u_1=0·19$ | $u_1=0·18$ |
| $u_2=0·19$ | $u_2=0·18$ |
| $q=\frac{1}{2}+r$ | $q=\frac{1}{2}+r$ |

| Alternative A. | Alternative B. |
|---|---|
| $r=0·10$ | $r=0·15$ |

It should be emphasized that $r$ cannot have values in the

* In Messrs. Dickson and Binks' work another system of notation is used. In our calculation the final results have been shortened so that all the parameters are given in two decimals only.