Dynamics of a System of Two Coupled MEMS Oscillators

Richard H. Rand[*,+], Alan T. Zehnder[*], B. Shayak[*]
[*]*Theoretical and Applied Mechanics, Sibley School of Mechanical and Aerospace Engineering,*
[+]*Department of Mathematics, Cornell University, Ithaca, NY, USA*

Summary. A model of a simplified MEMS device which has been previously shown to support a limit cycle [1] is used to model a pair of coupled MEMS oscillators. The stability and bifurcation of in-phase and out-of-phase modes is investigated.

Introduction

This work is concerned with a type of MEMS device in which a laser is used to measure the device's deformation via an interference pattern. As a side effect, the laser heats the device and affects the interference gap, resulting in a feedback loop which causes the device to vibrate in a limit cycle. A simplified model is considered which involves a second order ODE for the displacement z of the structure and a first order ODE for the temperature T due to the laser heating:

$$z'' + z = T, \qquad T' + T = z^2 - pz$$

where p is a parameter around 0.1. This system has been shown to support a limit cycle [1]. In the present work we use the foregoing to model a system of two coupled MEMS oscillators:

$$z_1'' + z_1 = T_1 + \alpha(z_2 - z_1), \qquad T_1' + T_1 = z_1^2 - pz_1$$

$$z_2'' + z_2 = T_2 + \alpha(z_1 - z_2), \qquad T_2' + T_2 = z_2^2 - pz_2$$

where α is a coupling constant.

Analysis

Simulation shows that this system admits an in-phase (IP) mode and an out-of-phase (OP) mode. To study these modes, we recast the system (1) in terms of sum and difference variables

$$X = z_1 + z_2, \quad Y = z_1 - z_2, \quad M = T_1 + T_2, \quad N = T_1 - T_2$$

To analyze the stability of these modes, we construct the linearized variational equations. First considering the IP mode, the variational equation for the difference variables can be expressed as a third order Floquet-type differential equation:

$$Y''' + Y'' + (1 + 2\alpha)Y' + (1 + 2\alpha + p + 2U(t))Y = 0$$

where $U(t)$ denotes the periodic limit cycle of the IP mode. If $p = 0.1$, a regular perturbation analysis yields a transition from stable to unstable when α is lowered beyond 0.04, a fact which is confirmed by numerical simulations.

For the OP mode, we introduce a small parameter ε and use Lindstedt's perturbation method. The resulting approximation for the OP mode is then inserted into the associated variational equation which yields the value of $\alpha = 0.88$ above which the OP mode loses stability, which is in good agreement with the numerically observed value of $\alpha = 0.83$.

Thus the IP mode is stable when $\alpha > 0.04$ and the OP mode is stable when $\alpha < 0.83$, which is to say that both the IP and OP modes are stable when α lies between 0.04 and 0.83. With both modes stable, we expect an unstable motion which has a stable manifold which lies on the boundary of the basins of attraction of the stable modes. Although this separatrix motion is unstable, we may seek it by numerically varying initial conditions, trying to find the point where the motion switches between the IP and OP modes. The resulting separatrix motion appears to be quasiperiodic. This situation is reminiscent of the dynamics of two coupled van der Pol oscillators [2], where a perturbation approximation for the separatrix motion was obtained.

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References

[1] R.H. Rand, A.T. Zehnder, B.Shayak, Analysis of a simplified MEMS oscillator, Proceedings of 9th European Nonlinear Dynamics Conference (ENOC 2017), June 25-30, 2017, Budapest, Hungary.

[2] D.W.Storti, R.H.Rand, Dynamics of Two Strongly Coupled Van der Pol Oscillators, *Int. J. Nonlinear Mechanics* 17:143-152 (1982).