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Dynamics of Two van der Pol Oscillators Coupled via a Bath

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In this work we study a system of two van der Pol oscillators, x and y, coupled via a "bath" z:

$$x'' - e(1-x^2)x' + x = k(z-x)$$
$$y'' - e(1-y^2)y' + y = k(z-y)$$
$$z' = k(x-z) + k(y-z)$$

Our motivation for studying this system comes from the chemistry of the eyes. There is experimental evidence that there exist circadian rhythms in each of our two eyes. These are periodic motions, limit cycles, with periods of approximately 24 hours. E.g. melatonin levels have been observed to vary periodically in this way. It has been conjectured that the visual system, which must operate over some 10 orders of magnitude in the course of night and day, has its sensitivity controlled through these oscillations by anticipating the changes in light intensity which occur at dusk and dawn.

It is naturally desirable that both eyes operate in phase. However, there is no direct connection between the two eyes. Instead, the eyes can influence each other by affecting the concentration of melatonin in the bloodstream. Alternatively, the eyes may be coupled via neural pathways in the brain. In this work we have chosen to model the former method of coupling between the eyes. x and y represent the concentrations of melatonin in each of the eyes, and z represents the concentration of melatonin in the bloodstream (here referred to as a bath.) We investigate the existence and stability of the in-phase and out-of-phase modes for parameters $e > 0$ and $k > 0$. To this end we use Floquet theory and numerical integration. Surprisingly, our results show that the out-of-phase mode exists and is stable for a wider range of parameters than is the in-phase mode. We also investigate the occurrence of other periodic motions by using the method of averaging for small values of e and k.