Applied Nonlinear Dynamics: Analytical, Computational and Experimental Methods. *By Ali H. Nayfeh and Balakumar Balachandran.* Wiley, New York, 1995. $64.95. xv+685 pp., cloth. ISBN 0-471-95348-6.

The modern theory of nonlinear dynamics and chaos had its beginnings in the nineteenth century work of Poincare. Although the subject continued to evolve since that time in the form of research papers, it was not until the first textbooks appeared in the early 1980s [1], [2] that it became easily accessible. Since that time there has been a virtual avalanche of books explaining the theory and application of nonlinear dynamics. Thus any new book on the subject must compete with all those that have come before, including those that are established references on the subject. The authors of the present work are well aware of this situation, and have listed in their preface 37 such books published between 1980 and 1994. Nevertheless they see their book as having a special appeal to readers who wish to see a treatment which combines analytical, experimental, and numerical approaches.

The book begins with an introductory chapter in which the usual definitions and concepts of dynamical systems are presented, such as maps versus flows, phase space, and various kinds of stability. This is followed by a chapter on the classification and bifurcation of equilibria, including the computation of stable, unstable, and center manifolds, as well as a discussion of structural stability. Chapter 3 deals with periodic solutions and Floquet theory, including the bifurcation of periodic solutions and their approximation (by the method of multiple scales). Chapter 4 treats quasiperiodic motions, torus flows, circle maps, and related topics such as Arnold tongues and the devil's staircase. The method of multiple scales is used to construct approximate quasiperiodic solutions to example systems. The next chapter, entitled "Chaos," offers an extensive review of recent results, including the period-doubling route to chaos, horseshoe maps, intermittency mechanisms, and quasiperiodic routes to chaos and crises. This chapter also includes a treatment of Melnikov's method applied to forced single degree of freedom systems. The chapter closes with a description of the Lorenz equations and Shilnikov's theorem. Chapter 6 deals with a variety of numerical methods including continuation methods for finding families of equilibria and periodic motions. Chapter 7 treats methods for characterizing data obtained from either experiments or numerical simulations, including the following concepts: embeddings, spectral analysis, autocorrelation functions, Lyapunov exponents, and fractal dimension. In the latter, the authors distinguish among capacity dimension, pointwise dimension, information dimension, correlation dimension, generalized correlation dimension, and Lyapunov dimension. Chapter 8, entitled "Control," includes a discussion of the control of chaos, featuring the Ott, Grebogi, and Yorke scheme. The book ends with an extensive bibliography involving 73 pages of references.

As an example of the level at which the book is written, take Melnikov's method, section 5.7. This method involves the evaluation of an integral which gives conditions on the parameters guaranteeing that chaos *cannot* occur. In preparation for Melnikov's method, the authors provide a description of homoclinic and heteroclinic tangles, which result from an intersection of stable and unstable manifolds. Then they discuss the direct use of numerical integration to determine the stable and unstable manifolds. Next they present the theory of the Melnikov integral, and they apply it to three examples of periodically forced single degree of freedom systems. In addition, they provide eight homework exercises relating to Melnikov's method. The emphasis here is on the computation of the Melnikov integral, as opposed to the theory behind it, i.e., the relationship between the existence of a horseshoe map and chaos. In fact, when the horseshoe is presented in section 5.3, no mention is made of its relationship to a shift on symbols, which has been shown to be the key to understanding its complicated behavior [1]. On the other hand, the treatment of the three examples is very clear, and can be understood by a less mathematically sophisticated reader than can [1]. The application of Melnikov's integral to two degree of freedom autonomous systems is omitted from the book (see [1]). Also omitted is Vakakis' direct computation of the stable and unstable manifolds by perturbation theory (see [3]).

For me the strength of the book is its interweaving of numerical results with perturbation methods. For example, in Chapter 3 an example system consisting of three first-order ODEs is shown by numerical integration to exhibit a variety of bifurcations as a single parameter is tuned. Then this system is treated by multiple scales, and again by center manifolds. The treatment of bifurcations throughout the book is particularly good, and I was especially interested in the discussion of intermittency mechanisms in Chapter 5. On the other hand, I was surprised to find the absence of a number of standard topics which might have been included. Missing from the book is KAM theory [1], [2], Lie transforms [3], and Chirikov's overlap criterion [3], [4]. Although the title of the book involves experimental methods, there is relatively little discussion of actual experiments; cf. [4].

This book would make an excellent textbook for a graduate course in nonlinear dynamics and chaos for engineers. It is written at a level which is accessible to such an audience, it covers a wide variety of topics, both classical and modern, and it contains a generous supply of homework exercises. It would also be a useful reference for a researcher.

REFERENCES

[1] J. Guckenheimer and P. Holmes (1983), *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, New York.

[2] A. J. Lichtenberg and M. A. Lieberman (1983), *Regular and Stochastic Motion*, Springer, New York.

[3] R. H. Rand (1994), *Topics in Nonlinear Dynamics with Computer Algebra*, Gordon and Breach, Langhorne, PA.

[4] F. C. Moon (1992), *Chaotic and Fractal Dynamics: An Introduction for Applied Scientists and Engineers*, Wiley, New York.

R. H. Rand
Cornell University