

# A Fluid-filled Spherical Shell Model of the Thermo-elastic Behaviour of Avian Eggs

S. K. UPADHYAYA\*; J. R. COOKE†; R. H. RAND‡

Egg shell breakage during hot water washing results from the differences in the volumetric expansion characteristics of the shell and its contents. Upon heating, the yolk and albumen expand while the enclosed volume of a hollow egg shell has an anomalous shrinkage response.

An incompressible liquid and compressible gas-filled elastic, spherical shell model is developed to examine the thermoelastic response to hot water washing. Two bounding cases of a shell which is (1) completely impervious and (2) completely porous to air cell leakage are considered.

Egg shell strength is known to decrease with increasing temperature. When combined with internal pressures induced by the washing process, a temporary, but appreciable reduction in the shell's capacity to withstand mechanical loads is predicted. Increases in shell porosity, air cell volume and egg shell size decrease the thermally produced mechanical stress. On the other hand, shell thickness does not appreciably influence the *thermally* induced stress.

## 1. Introduction

Egg shell breakage resulting from the hot water washing process in commercial egg production results in a substantial economic loss to the poultry industry. In this paper we examine the thermally induced stresses in egg shells resulting from this washing process. Specifically, we consider the role of the volumetric expansion of the fluid (yolk and albumen) and gas (air cell) enclosed by a porous shell. The egg has a remarkable thermal property: upon heating, the volume enclosed by the shell actually tends to decrease slightly (due to a negative coefficient of thermal expansion for the shell material in the plane of the shell surface) while the contents tend to expand, resulting in a pressure-induced stress in the shell. For an idealized spherical shape we explore two extreme cases which bound the case of greatest interest:

- (1) an impervious shell which prevents the escape of air contained in the air cell;
- (2) a completely porous shell which does not restrain the escape of the air in the air cell.

## 2. Review of literature

In 1981 the total production of hens' eggs in the U.S.A. amounted to 5800 million dozen eggs resulting in a gross income of \$3663 million.<sup>1</sup> Egg shell breakage is a major source of economic loss to the egg industry. Eggs cracked on the farm<sup>2</sup> ranged from 0.1 to 10% of production with an average of about 2.8%. Morris *et al.*<sup>3</sup> estimated breakage on the farm in the range of 0 to 8.89% for gathering and 0 to 7.78% for washing and grading. The total average breakage was 7.3% during processing<sup>4</sup> at grading stations. In terms of dollars, this represented an annual loss<sup>2, 5</sup> of \$60 million. Reduction of damage by even a percentage point means millions of dollars saved.<sup>6</sup> Although the broken eggs are salvaged for use in commercial bakeries for processing (e.g. cake mixes), the economic return is reduced.

The main cause of eggshell breakage is the repeated mechanical and/or thermal loadings to which eggs are subjected before reaching the consumer. Mechanical loadings consist of point-impacts of varying intensity which may occur at the time of laying or during processing, packing

\*Department of Agricultural Engineering, University of California, Davis, California, U.S.A.

†Department of Agricultural Engineering, Cornell University, Ithaca, New York, U.S.A.

‡Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York, U.S.A.

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## NOTATION

$E$	Young's modulus for the shell material, Pa
$P_o$	absolute pressure of air in the air cell, Pa
$R$	radius of the egg shell, m
$T$	$T_w - T_o$ , K
$T_w$	absolute temperature of the wash water, K
$T_o$	initial absolute temperature of wash water, K
$V_a$	volume of air in the air cell, m <sup>3</sup>
$V_l$	volume of yolk and albumen, m <sup>3</sup>
$V_o$	initial volume of air in the air cell, m <sup>3</sup>
$V_s$	volume enclosed by the egg shell, m <sup>3</sup>
$c_p$	specific heat of the yolk and albumen at constant pressure, kJ kg <sup>-1</sup> K <sup>-1</sup>
$k$	thermal conductivity of the yolk and albumen, kJm <sup>-1</sup> s <sup>-1</sup> K <sup>-1</sup>
$n$	$n$ th harmonic
$p$	internal pressure, Pa
$r$	radial distance from the origin, m
$r_o$	outer radius of albumen boundary, m
$t$	time, s
$\hat{t}$	thickness of the egg shell, m
$u$	radial displacement of the shell, m
$v$	change in volume of the air cell, m <sup>3</sup>
$\Delta$	a small change
$\hat{a}$	$k/Pc =$ thermal diffusivity, m <sup>2</sup> s <sup>-1</sup>
$\beta$	$1 - 6/\pi^2 \sum \exp(-\lambda_n t)/n^2$
$\gamma$	volume expansion coefficient for the yolk and albumen, m <sup>3</sup> m <sup>-3</sup> K <sup>-1</sup>
$\delta$	ratio of air cell volume to total enclosed volume of the egg shell
$\epsilon_{th}$	thermal strain in the shell in the circumferential direction, m m <sup>-1</sup>
$\epsilon_\theta$	total circumferential strain, m m <sup>-1</sup>
$\xi$	$1 - \delta$
$\rho$	density of albumen and yolk, kg m <sup>-3</sup>
$\sigma_\theta$	circumferential stress in the shell, Pa
$\sigma_\varphi$	meridional stress in the shell, Pa
$\nu$	Poisson's ratio of shell material
$\varphi$	temperature field, K

and transport. Thermal loads occur due to washing, candling and cooling. Several investigations of shell failure due to mechanical loading have been conducted,<sup>2</sup> but few have considered the effect of thermal loading on egg shell breakage. The importance of the thermal loading is clear from the following.

Orr *et al.*<sup>7</sup> reported that considerable shell damage occurred during washing, grading and packing. This damage involved cracks produced by internal pressure, and by impacts. The pressure might be due to either mechanical loading and/or thermal shock. Morris, Harwood and Brooks<sup>3</sup> estimated that the breakage during washing, grading and packing averaged about 3.6%. During loading and washing, the loss was estimated to be about 2%. Anderson, Carter and Jones<sup>8</sup> found a 3% loss after the arrival of the eggs at the plant and before candling. The shell strength was found to decrease with increasing temperature.<sup>9-11</sup> Nelson and Henderson<sup>12</sup> cited a study in which the thermal cracks in the egg shell were found to increase with an increase

in the temperature. Carter<sup>13</sup> conjectured that shell splitting during boiling was due to the expansion of the internal contents of the shell.

Manceau and Henderson<sup>14</sup> found by a finite element analysis that the transient thermal stress alone would not crack a hollow egg shell. However, they conjectured that the rise of internal pressure due to the differential expansion of the shell and its contents could play an important role. Thermal cracking normally occurred as a latitudinal crack, similar to the cracks from internal pressure. Mohsenin<sup>15</sup> concluded that the thermal stress alone does not crack the egg shell unless the egg has been subjected to a sufficiently high mechanical stress. Rehkugler<sup>16</sup> found that the tensile failure stress decreased by 17 MPa when the egg was heated for a few minutes using hot water. He found that the thermal expansion coefficient of the shell material did not change with temperature in the range 25 to 68°C; he also found that the thermal gradient within the shell material was small. His approximate calculations showed that the internal pressure could become great enough to cause fracture of the egg shell.

### 3. Objectives

In the present paper we mathematically examine the hypothesis that thermal cracking of eggs is caused by the increase in internal pressure which results from the differential expansion of the shell and its contents when subjected to hot water washing.

The specific objectives of this study were:

- (1) to investigate the transient stress in the egg shell during thermal loading using a simplified fluid-filled spherical shell model (see Fig. 1);
- (2) to investigate the effect of egg shell size, thickness and air cell size on the thermally induced stress in the egg shell.

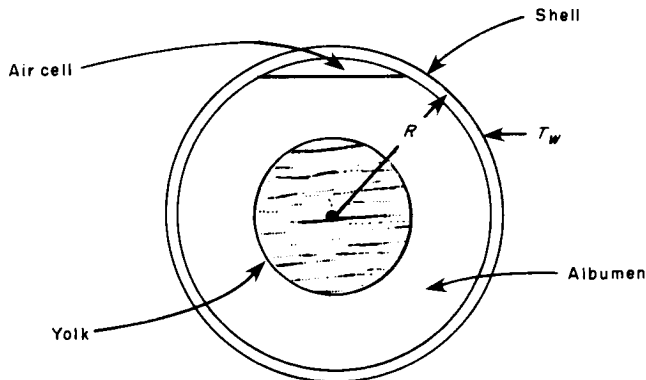


Fig. 1. Simplified spherical model for the egg shell

### 4. Mathematical modelling

The egg shell is a thin, approximately ovaloidal shell composed mainly of calcite. The shell is both elastically and thermally orthotropic. It contains an air cell at the blunt end and the rest of the shell is filled with a whitish semi-opaque liquid called albumen. Within the albumen is situated a yellowish spherical yolk. The air cell is enclosed between a shell membrane and an egg membrane. In order to model the thermal behaviour of such a fluid-filled shell we shall assume:

- (1) the egg shell is spherical in shape (a more general ovaloidal shell is considered in a sequel to this paper<sup>17</sup>);

- (2) the shell material is elastically isotropic;
- (3) the liquid albumen and yolk are incompressible;
- (4) air in the air cell obeys the ideal gas law;
- (5) the thermal anisotropy of the shell material is restricted to the radial direction (spherically symmetric orthotropy).

Manceau and Henderson<sup>6</sup> reported that the shell has a negative coefficient of thermal expansion in the tangential direction (i.e. in the plane of the shell surface). Therefore, upon heating, the shell will shrink in size. However, the albumen and yolk expand upon heating. Both effect an increase in the internal pressure on the shell when heated. As a result the air cell decreases in size and some air may escape through the shell and membrane pores. The internal pressure induces a tensile stress in the shell. Thus the thermal loading of the egg shell involves the following three problems linked to each other by a volume constraint.

- (1) Thermoelasticity of the spherical shell
- (2) Thermal expansion of the albumen and yolk
- (3) The compression and escape of air in the air cell.

Let  $V_s$  be the volume enclosed by the egg shell,  $V_l$  be the volume of albumen and yolk and  $V_a$  be the volume of air in the air cell; then,

$$V_s = V_l + V_a$$

For small changes in volume

$$\Delta V_s = \Delta V_l + \Delta V_a \quad \dots (1)$$

where,

$\Delta V_s$  = the change in volume enclosed by the shell,  $m^3$

$\Delta V_l$  = the change in the volume of the albumen and yolk,  $m^3$ , and

$\Delta V_a$  = the change in the volume of the air in the air cell,  $m^3$ .

In the following discussion we consider each of these volume changes separately.

#### 4.1. Change in the enclosed volume of the shell, $\Delta V_s$

The radial expansion of a thin spherical shell subjected to an internal pressure  $p$  and a uniform temperature field  $T$  may be obtained by the membrane theory of shells of revolution as follows.

We assume that (1) the egg shell is spherical in shape; (2) the shell material is elastically isotropic; (3) the direction of thermal anisotropy is restricted to the radial direction; (4) the thickness of the shell material is small compared to the radius of the shell, and there are no abrupt changes in curvature, so that bending stresses can be neglected; (5) the thermal conductivity of

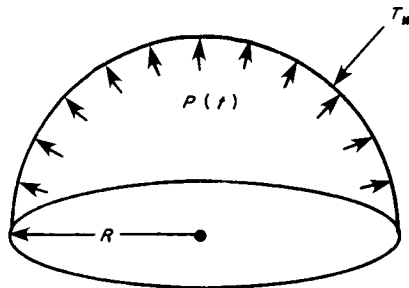


Fig. 2. Force diagram for a spherical shell under internal pressure

the shell material is so high that it reaches temperature equilibrium with the wash water almost instantaneously.<sup>16</sup> The shell is assumed to be in intimate contact with the hot water, i.e. there is no film effect; (6) the thermal stress due to the temperature gradient in the shell is negligible. The assumption (4) of negligible bending stresses is valid for any uniformly distributed or smoothly varying load, regardless of shell geometry, provided there are no abrupt changes in shell curvature. As a consequence, we may apply the simpler so-called 'membrane theory' for shells. Then the following relations hold for the membrane stresses<sup>18</sup> (see Fig. 2):

$$\sigma_{\theta} = \sigma_{\phi} = \frac{pR}{2t} \quad \dots (2)$$

where

- $\sigma_{\theta}$  = circumferential stress, Pa
- $\sigma_{\phi}$  = meridional stress, Pa
- $p$  = internal pressure, Pa
- $R$  = radius of the spherical shell, m
- $t$  = thickness of the shell, m

The circumferential membrane strain in the shell is given by

$$\varepsilon_{\theta} - \varepsilon_{th} = (\sigma_{\theta} - \nu\sigma_{\theta})/E \quad \dots (3)$$

where

- $\varepsilon_{\theta}$  = circumferential elastic strain,  $m m^{-1}$
- $\varepsilon_{th}$  = circumferential thermal strain,  $m m^{-1}$
- $E$  = Young's modulus of elasticity, Pa
- $\nu$  = Poisson's ratio

Thermal strain is given by

$$\varepsilon_{th} = \alpha T \quad \dots (4)$$

where

- $\alpha$  = coefficient of linear expansion of the shell material,  $mm^{-1} \text{ } ^\circ C^{-1}$
  - $T$  = change in temperature of the shell,  $^\circ C$
- and the circumferential elastic strain is given by

$$\varepsilon_{\theta} = \frac{u}{R} \quad \dots (5)$$

where

- $u$  = radial displacement,  $m$
- Therefore, from Eqns (2)–(5), we obtain

$$\frac{u}{R} - \alpha T = \frac{pR}{2tE} (1 - \nu)$$

or

$$u = R \left[ \alpha T + \frac{pR}{2tE} (1 - \nu) \right] \quad \dots (6)$$

Therefore, the change in the enclosed volume of the shell becomes

$$\begin{aligned} \Delta V_s &= \frac{4}{3}\pi[(R+u)^3 - R^3] \\ &\simeq 4\pi R^2 u \\ &= 4\pi R^3 [aT + pR(1-\nu)/2E\hat{t}] \quad \dots (7) \\ \Delta V_s &= 3V_s [aT + pR(1-\nu)/2E\hat{t}] \end{aligned}$$

where the higher order terms in  $u$  have been neglected.

4.2. *Change in the volume of the albumen and yolk,  $\Delta V_1$*

The change in the volume of albumen and yolk is obtained by a transient thermal analysis of the internal contents as described below:

Although the shell is assumed to be spherical, the albumen boundary is not spherical due to the presence of the air cell at one end (see Fig. 1). However, as the air cell is normally small in size, the boundary between the albumen and shell can be approximated by a sphere (see Fig. 3).

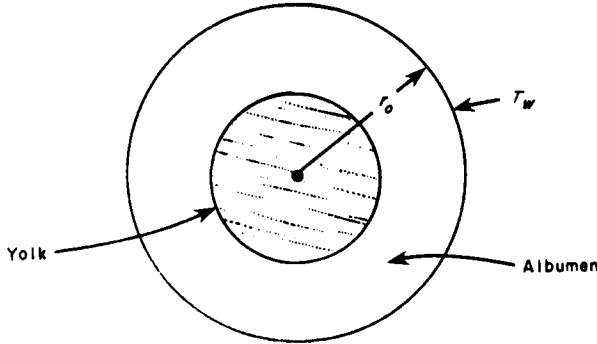


Fig. 3. Model of thermal response of the albumen

If the albumen is in contact with the shell except for a small sector at the air-cell interface, the surface temperature will reach wash water temperature ( $T_w$ ) almost instantaneously everywhere except at the air cell-shell interface. The air-albumen interface will be at a lower temperature since air is a poor heat conductor. However, the most critical case, in terms of the albumen and yolk expansion and high internal pressure, corresponds to an air-albumen interface temperature of  $T_w$ . Consequently we assume the albumen boundary to be spherical and to be at the uniform temperature  $T_w$ .

The differential equation for thermal conduction for the above spherically symmetric case becomes (Sokolnikoff and Redheffer):<sup>19</sup>

$$\varphi_t = \hat{\alpha} \left[ \frac{1}{r^2} (r^2 \varphi_r)_r \right] \quad \dots (8)$$

where

$\varphi$  = temperature, K

$\hat{a} = \frac{k}{\rho c}$  = thermal diffusivity,  $m^2 s^{-1}$

$k$  = thermal conductivity,  $W/(m K)$

$c_p$  = specific heat,  $J/(kg K)$

$r$  = radial distance from the origin, m

$t$  = time, s

and where subscripts  $r$  and  $t$  represent partial differentiation. The temperature of the outer surface is equal to the water temperature. Therefore, the boundary condition is

$$\varphi(r_o, t) = T_w.$$

The shell has an initial uniform temperature. Therefore, the initial condition is

$$\varphi(r, 0) = T_o.$$

Using the method of separation of variables we obtain the temperature

$$\varphi(r, t) = T_w + \frac{2(T_w - T_o)}{r} (r_o) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-\lambda_n t} \sin\left(\frac{n\pi}{r_o}\right) r \quad \dots (9)$$

where

$$\lambda_n = \hat{a} \left(\frac{n\pi}{r_o}\right)^2$$

Since the yolk and albumen have nearly equal thermal properties, average values are used (see Table 1). The change in the volume,  $\Delta V_i$  of the albumen is given by:

$$\Delta V_i = 4\pi\gamma \int_r^{r_o} \rho^2 (\varphi - T_o) dr \quad \dots (10)$$

where

$\gamma$  = volume expansion coefficient for yolk and albumen,  $m^3 m^{-3} K^{-1}$

$$\int_r^{r_o} 4\pi\gamma r^2 \left[ T_w - T_o + \frac{2(T_w - T_o)}{\pi r} (r_o) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-\lambda_n t} \sin\left(\frac{n\pi}{r_o}\right) r \right] dr$$

Evaluation of these integrals gives

$$\Delta V_i = V_i \gamma T \left[ 1 - \frac{6}{\pi^2} \sum \frac{e^{-\lambda_n t}}{n^2} \right] \quad \dots (11)$$

where

$$T = T_w - T_o$$

$$V_i = \frac{4}{3} \pi r_o^3$$

TABLE I

## Physical and geometrical parameters used in the model

Parameter	Value	Reference
<i>Shell</i>		
(1) Young's modulus ( $E$ )	$4.69 \times 10^{10}$ Pa	6
(2) Poisson's ratio ( $\nu$ )	0.307	
(3) Linear thermal expansion ( $\alpha$ )	$-4.365 \times 10^{-6}$ m m <sup>-1</sup> K <sup>-1</sup>	
<i>Yolk and albumen</i>		
(4) Density ( $\rho$ )	1035 kg/m <sup>3</sup>	20
(5) Specific heat ( $c$ )	3.56 kJ kg <sup>-1</sup> K <sup>-1</sup>	
(6) Thermal conductivity ( $k$ )		
(a) yolk	$3.37 \times 10^{-4}$ kJ m <sup>-1</sup> s <sup>-1</sup> K <sup>-1</sup>	21
(b) albumen	$5.57 \times 10^{-4}$ kJ m <sup>-1</sup> s <sup>-1</sup> K <sup>-1</sup>	22
weighted average*	$4.56 \times 10^{-4}$ kJ m <sup>-1</sup> s <sup>-1</sup> K <sup>-1</sup>	
(7) Thermal diffusivity ( $\hat{\alpha} = \sqrt{\frac{k}{C\rho}}$ )	$1.24 \times 10^{-7}$ m <sup>2</sup> s <sup>-1</sup>	
(8) Thermal expansion coefficient	$4.652 \times 10^{-4}$ m <sup>3</sup> m <sup>-3</sup> K <sup>-1</sup>	16 and lab tests conducted by the authors
<i>Average geometry</i>		
(9) Radius of the egg shell ( $R$ )	23.2 mm	20
(10) Thickness of the egg shell ( $\hat{t}$ )	$3.16 \times 10^{-4}$ m	
(11) Volume, percentage of the enclosed volume		
(a) yolk and albumen	99.2%	20
(b) air cell	0.8%	
<i>Ambient conditions</i>		
(12) Atmospheric pressure ( $P_o$ )	101325 Pa	
(13) Temperature ( $T_o$ )	293 K	

\*This average is found<sup>22</sup> using the formula for two thermal conductors arranged in series. Note that this technique is not accurate as it applies for steady state conditions with a straight boundary. However, since yolk and albumen do not differ in thermal conductivity by an order of magnitude this approximate method is thought to be adequate in this case

4.3. Change in the volume of the air cell,  $\Delta V_a$ 

Let  $P_o$ ,  $V_o$ ,  $T_o$  respectively be the pressure, volume and temperature of the air in the air cell under ambient conditions. Let the pressure, volume, and temperature change be  $p$ ,  $v$  and  $T$  respectively at some time  $t$  during the washing process. Then from the ideal gas law

$$\frac{P_o V_o}{T_o} = \frac{(P_o + p)(V_o + v)}{T_o + T} \quad \dots (12)$$

As  $\Delta V_a = v$ ,

$$\Delta v_a = v = V_o \left[ \left( \frac{P_o}{P_o + p} \right) \left( \frac{T_o + T}{T_o} \right) - 1 \right] \quad \dots (13)$$

Eqns (7), (11) and (13) can be substituted into Eqn (1) to obtain  $p$  as follows:



Let the volume of the air cell,  $V_a$ , be a fraction  $\delta$  of the total enclosed initial volume of the shell,  $V_s$ ,

$$V_o \equiv V_a = \delta V_s \quad \dots (14)$$

and

$$V_l = (1 - \delta)V_s$$

Therefore,

$$3V_s[aT + pR(1 - \nu)/2E\hat{t}] = \left[ \left( \frac{P_o}{P_o + p} \right) \left( \frac{T_o + T}{T_o} \right) - 1 \right] \delta V_s + (1 - \delta)V_s \beta\gamma T \quad \dots (15)$$

which upon simplification becomes

$$(P_o T - p T_o) \delta = (P_o + p) T_o \left[ (3a - \xi\beta\gamma)T + \frac{3R(1 - \nu)}{2E\hat{t}} p \right] \quad \dots (16)$$

where,

$$\beta = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \left( \frac{e^{-\lambda_n t}}{n^2} \right)$$

$$\xi = 1 - \delta$$

Eqn (16) is a quadratic in  $p(t)$  and can be solved for  $p$  at a given time  $t$ .

However, it is quite possible that some amount of the air diffuses out. Inclusion of the diffusion effects would further complicate the model, so for simplicity only two distinct bounding cases are considered.

Case 1. The membrane is impermeable to air (i.e. air does not leak out at all). This case corresponds to Eqns (13) and (16).

Case 2. The membrane is completely permeable to air (i.e. air leaks out easily). In this case air does not compress at all. Therefore, until the air completely leaks out, there is no pressure build-up inside the shell. Therefore,

$$p = 0 \text{ if } \Delta V_l - \Delta V_s < \Delta V_a = \delta V_s$$

Once all the air has left the shell the pressure,  $p$ , can be found from the constraint equation

$$\Delta V_l - \Delta V_s = V_a = \delta V_s$$

$$\xi\gamma\beta T - [3aT + 3pR(1 - \nu)/2E\hat{t}] = \delta$$

or

$$p(t) = 2E\hat{t}[(\xi\gamma\beta - 3a)T - \delta]/[3R(1 - \nu)] \quad \dots (17)$$

The membrane stress,  $\sigma_t$ , in the shell due to the thermal load is given by

$$\sigma_t = pR/2\hat{t} \quad \dots (18)$$

Eqns (16), (17) and (18) were evaluated for various wash water temperatures using physical parameters taken from the literature and listed in Table 1.

## 5. Results and discussion

### 5.1. Transient thermal stress on the egg shell

Fig. 4 shows the transient stress levels for an egg having the physical and geometrical parameters given in Table 1 and subjected to a wash water temperature of 45°C. The eggs were assumed to be stored at 20°C prior to wash water treatment. Note that in the first 15 min stresses

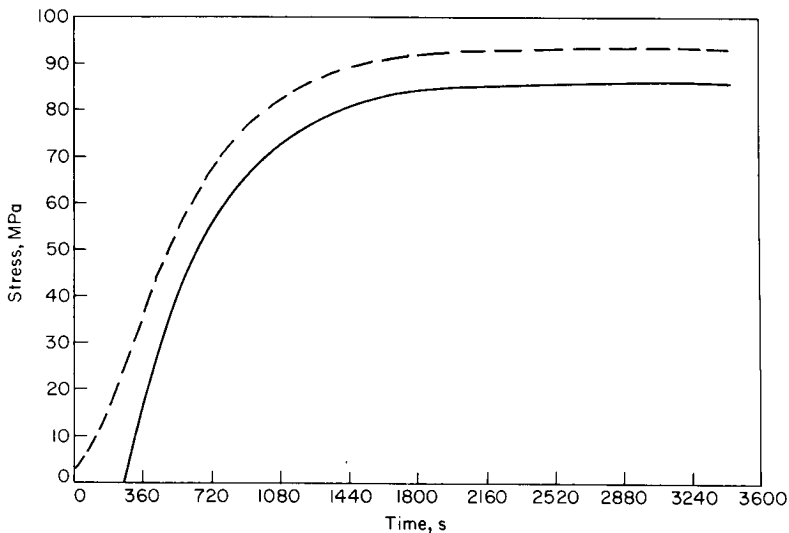


Fig. 4. Thermal stresses in an egg shell stored at 20°C when washed with water at 45°C. —, Air leaks out; - - -, air stays in

in the shell reach about 80% of the steady state value. Within half an hour the stresses asymptotically approach the steady state value. A close examination of the curve suggests that there is an inflection point about 3 min after the start of washing. This phase is due to the compression of the air cell. Note that if the air leaks out completely (as shown in the lower curve), this phase does not occur. As expected, the internal pressure and the stresses are lower at all times if the air leaks out completely. However, the difference between these two cases decreases as the wash water temperature increases from 35 to 55°C. At higher wash water temperatures the air in the air cell does not influence the stresses in the shell appreciably, except in the first few seconds (Figs 4, 5 and 6).

Rehkgler<sup>16</sup> reported a 62% decrease in the mean tensile stress at failure between eggs stored and tested at room temperature (stress at failure = 54.1 MPa) compared with eggs stored at 5°C but immersed in a water bath at 45.6°C prior to testing (stress at failure = 37 MPa). The time between removal from storage and fracturing of the egg was of the order of minutes (*c.* 3 min). The eggs had a mean length of 58.2 mm and a mean width of 42.4 mm. The mean thickness of the

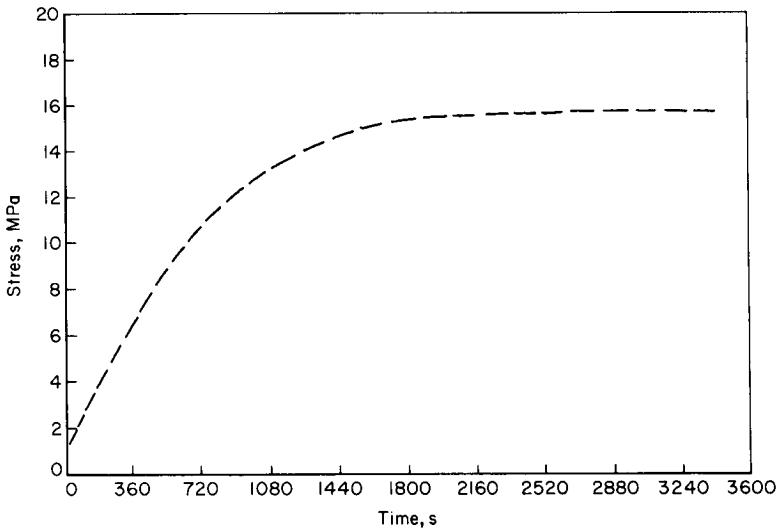


Fig. 5. Thermal stresses in an egg shell stored at 20°C when washed with water at 35°C. —, Air leaks out (coincides with time axis); ---, air stays in

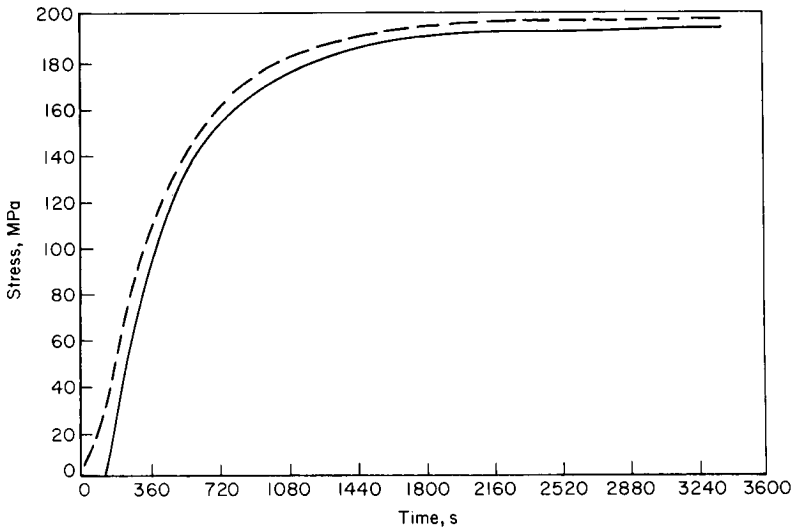


Fig. 6. Thermal stresses in an egg shell stored at 20°C when washed with water at 55°C. —, Air leaks out; ---, air stays in

shell was 0.343 mm. Our model predicts a tensile stress of 13.7 MPa at 3 min and 16.9 MPa at 5 min, for a spherical egg of the same volume and thickness if no air leaks out. The size of the air cell which results in these stress levels is 0.15 ml. Romanoff and Romanoff<sup>20</sup> reported an air cell volume of 0.1 to 0.2 ml immediately after appearance. The average size of a fully developed air cell,<sup>20</sup> however, is 0.4 ml. A larger air cell volume leads to lower stress levels in the shell (Fig. 9).

In a nearby poultry industry we found that an egg typically takes 2 to 3 min to pass through the washing stage with a wash-water temperature of 45°C. Our simulations with 45°C wash-water predicts a stress level of 14.4 MPa in 3 min if no air leaks out. In the ambient air the tensile failure

stress of egg shell<sup>16</sup> ranges from 17 to 54 MPa. Thus, washing in 45°C water for 3 min can result in a range in pre-stress of between 14.4/17 and 14.4/54, or 85% and 27%. Therefore, while passing through the wash water an egg may be pre-stressed by at least 27% of its tensile failure stress. Any subsequent mechanical loading during grading and packing may increase the resultant tensile stress at the lower surface due to the bending effects of the mechanical loading of the shell and may induce breakage.

Note that the 'effective decrease' in the strength due to this thermal pre-stressing is only temporary. As the egg cools, the internal pressure and hence the stress imposed on the shell will

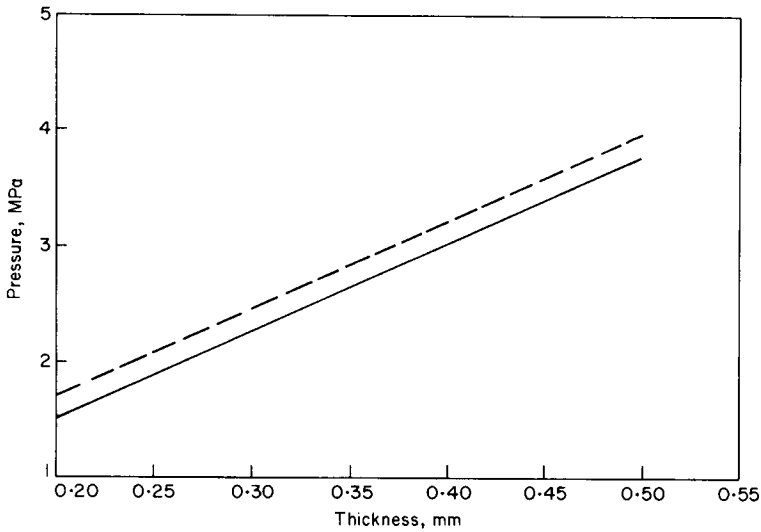


Fig. 7. Effect of egg shell thicknesses on the internal pressures. —, Air leaks out; ---, air stays in

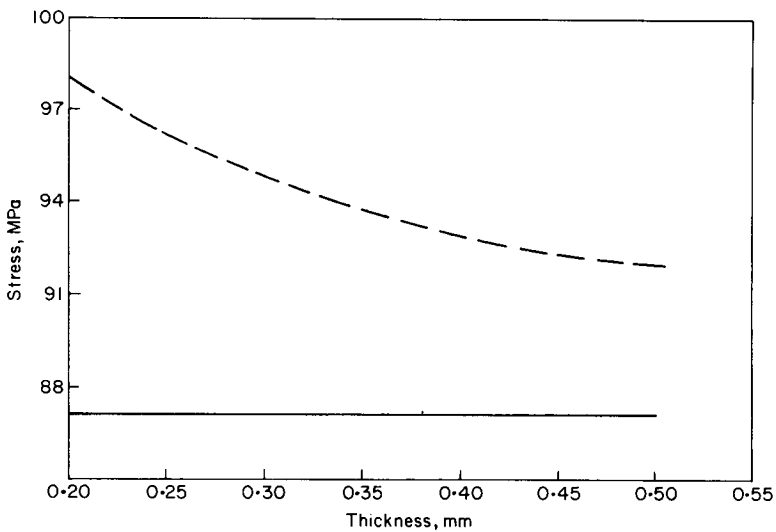


Fig. 8. Effect of egg shell thicknesses on the thermal stresses. —, Air leaks out; ---, air stays in

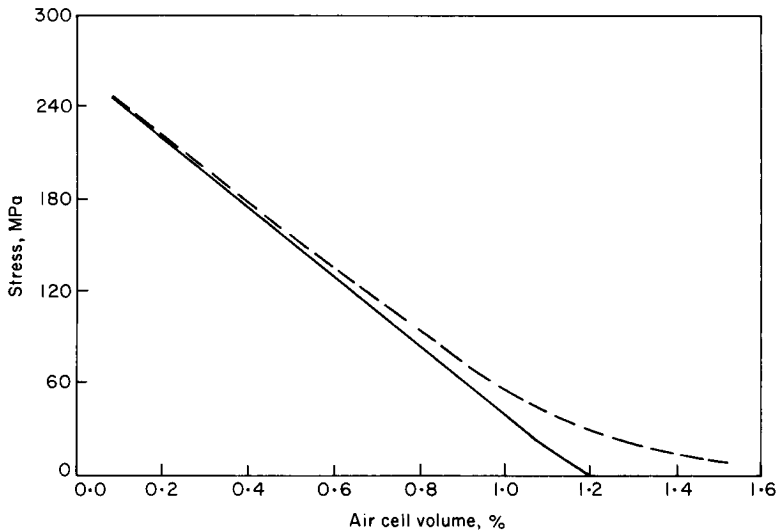


Fig 9. Effect of air cell volume on thermal stresses. —, Air leaks out; ---, air stays in

be relieved. In some poultry industries the eggs are cooled before candling, to make the blood spot detection easier. This practice may also assist in reducing breakage. If this is not done, a short cooling stage placed after the washing and drying stage but before the candling stage should reduce egg shell breakage.

When the conveyors are overloaded a micro-switch turns off the conveyor. During this period eggs in the washer might stay there for 10 to 15 min. Even though the hot water jets are turned off, the air is saturated with hot vapour and can introduce thermal stresses. For parameters shown in Table 1 and 45°C wash water temperature the stress can be as high as 59 MPa in 10 min. If all the air from the cell leaks out the stress can be as high as 47 MPa. If the duration is extended to 15 min, the stress can be as high as 76 MPa if the air does not leak out or 67 MPa if the air leaks out. In any case, if the pressure is not relieved due to the seepage of albumen through the shell-pores, hairline pressure cracks may result. Therefore, as a precaution it is advisable to avoid holding the eggs in the washer for an extended period of time.

The time dependence of the thermal stress is due to the thermal expansion of the albumen and yolk with time (Eqn 11). As Eqn (11) does not contain a thickness term, the effect of the thickness on the thermal stress may be determined using a steady state analysis. The effect of egg shell thickness on the internal pressure is shown in Fig. 7. We see that as the thickness of the shell increases, the internal pressure also increases. However, the thermal stress decreases slightly (6.3% decrease for 150% increase in thickness) as the thickness increases, for the case where none of the air cell leaks out (Fig. 8). If the air in the cell leaks out, shell thickness does not influence the thermal stress in the shell (Fig. 8). This is in contrast to the stresses due to mechanical loading which depend on thickness. However, if we were to consider the actual geometry of the egg (modelled, say, by an ovaloid), thickness may slightly influence the thermal stress levels. It is to be noted that consideration of such a geometry makes the problem extremely difficult; an analytical solution is probably not possible.

Recently, a finite element analysis of this problem has been completed. Results from the analytical model presented here, using a simplified spherical geometry, agree surprisingly well with predictions made using the finite element method. Comparison of results indicates that the internal pressure may be adequately predicted by our simplified spherical model, although maximum tensile stresses developed are *greater* for more realistic ovaloid shapes (all other factors

constant). Thus the trends in the relations among various shell parameters, and their practical implications, are obtained with considerable less computational effort than that required by the finite element solution.<sup>17</sup>

The effect of air cell volume on thermal stress in the shell can also be inferred from the steady state analysis, since the air cell volume is very small compared to the enclosed volume of the shell and since air in the air cell is assumed to be in thermal equilibrium with the surrounding water-bath. The variation of thermal stress as the percentage air cell volume is changed is shown in Fig. 9. The thermal stress decreases as the percent air cell volume increases. Note that there is hardly any difference in the induced thermal stress if the air cell volume is small whether or not the air in the air cell leaks out. As the percentage air cell volume increases the difference in the thermal stresses between the two cases increases until the thermal stress in the shell becomes zero for the case for which the air in the air cell leaks out.

The effect of the size of the egg (radius) on the thermal stress is determined using the transient analysis since Eqn (11) contains radius  $r_0$  as a parameter in  $\lambda_n$ . Our model predicts a decrease in the thermal stress as the radius is increased for either the case in which the air in the air cell does or does not leak out.

### Conclusions

- (1) The model predicts that if the air is permitted to leak out of the shell, the thermal stresses will be lower than if the air does not leak out.
- (2) As the wash water temperature is increased, the difference in thermal stress between the case where the air completely leaks out and where it does not, becomes smaller.
- (3) Washing an egg in warm water can temporarily, but appreciably decrease the effective strength of the egg. This implies a higher chance of breakage during candling and packing, if the egg is not cooled. Prolonged exposure of eggs to hot water should be avoided.
- (4) Egg shell thickness does not greatly influence the *thermal* stress in the shell.
- (5) As the air cell volume increases, the induced thermal stress decreases.
- (6) An increase in egg radius leads to a decrease in the thermal stress in the shell.

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