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SPINUP THROUGH RESONANCE OF ROTATING UNBALANCED SYSTEMS WITH LIMITED TORQUE

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Abstract

When the platform of a dual-spin spacecraft (s/c) is despun from an all spun condition, several capture phenomena can occur which prevent the s/c from reaching its desired terminal state. These include minimum energy trap [1, 2]; nutation resonance [1, 3]; and precession phase lock [4]. This paper focuses on the capture dynamics of precession phase lock (PPL). A s/c model of the minimum complexity needed to illustrate PPL is studied. The s/c platform is assumed to be axially symmetric and balanced, and the rotor is assumed to be axially symmetric and dynamically unbalanced. The despin motor is assumed to provide a small constant torque. During PPL, dynamic effects due to the unbalance cause secular growth in the s/c cone angle. A method is developed by which the final cone angle may be estimated analytically as a function of the s/c mass properties and the despin motor torque magnitude. The calculations needed to obtain the estimate are relatively simple. The estimates are valid for cases in which the s/c experiences cone angle growth of up to 30° as it passes through PPL. Computer simulations are used to compare cone angle estimates with exact values for several cases.

I. Introduction

An idealized dual-spin s/c model with the minimum complexity required for precession phase lock (PPL) to occur is shown in Figure 1. The platform (body A) is axially symmetric and balanced, and the rotor (body B) is axially symmetric but dynamically unbalanced. The platform and rotor share a common axis, \hat{b}_3 , referred to as the "spin" axis,

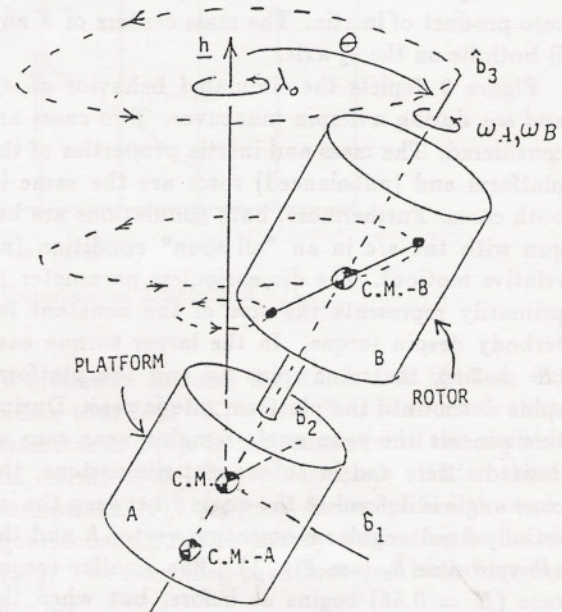


Figure 1: Idealized Unbalanced Dual-Spin S/C

about which relative rotation is possible. When the s/c is placed in orbit, the two bodies A and B rotate together with zero relative rate. Then an internal torque motor is turned on to reduce the angular velocity of the platform (A) and increase the angular velocity of the rotor (B). This maneuver is called a despin maneuver.

Let the inertial angular velocities of A and B be expressed as

$$\underline{\omega}^A = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_A \hat{b}_3 \quad (1)$$

$$\underline{\omega}^B = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_B \hat{b}_3 \quad (2)$$

where $\hat{b}_1, \hat{b}_2, \hat{b}_3$ are unit vectors fixed in B as shown in Fig. 1. Further, let I^A and I^B be the inertia matrices for A and B, respectively, for axes parallel to $\hat{b}_1, \hat{b}_2, \hat{b}_3$ and passing through the system c.m. I^A and I^B are assumed to have the form

$$I_A = \begin{bmatrix} I_{11}^A & 0 & 0 \\ 0 & I_{11}^A & 0 \\ 0 & 0 & I_{33}^A \end{bmatrix} \quad (3)$$

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$$I_B = \begin{bmatrix} I_{11}^B & 0 & I_{13}^B \\ 0 & I_{11}^B & 0 \\ I_{13}^B & 0 & I_{33}^B \end{bmatrix} \quad (4)$$

From (3) and (4) we see that A is symmetrical and dynamically balanced for $\hat{b}_1, \hat{b}_2, \hat{b}_3$. B is also symmetrical in the sense that the moments of inertia for \hat{b}_1 and \hat{b}_2 are equal. However, B is dynamically unbalanced due to the presence of a non zero product of inertia. The mass centers of A and B both lie on the \hat{b}_3 axis.

Figure 2 depicts the simulated behavior of ω_A and ω_B during a despin maneuver. Two cases are considered. The mass and inertia properties of the platform and (unbalanced) rotor are the same in both cases. Furthermore, both simulations are begun with the s/c in an "all spun" condition (no relative motion). The dimensionless parameter K primarily represents the size of the constant interbody despin torque. In the larger torque case ($K = 2.0$), the rotor spins up and the platform spins down until the platform rate is zero. During this process the cone angle remains near zero as desired. Here and in subsequent discussions, the cone angle is defined as the angle θ between the inertially fixed angular momentum vector \underline{h} and the s/c spin axis \hat{b}_3 (see Fig. 1). The smaller torque case ($K = 0.56$) begins as before, but when the rotor rate reaches the inertial free precession rate, the behavior changes markedly. Both the rotor rate and the platform rate decrease toward zero and the spin axis momentum is transferred into transverse coning motion. By the time the platform rate reaches zero, the cone angle has increased to a large value and the s/c is essentially tumbling end over end. The behavior exhibited in the second case is referred to as precession phase lock (PPL) because rapid cone angle growth is initiated when the rotor rate ω_B is approximately equal to the free precession rate of the s/c. During the period of cone angle growth, the rotor remains nearly fixed relative to the plane containing \underline{h} and \hat{b}_3 .

The above two cases represent two extremes. The first case occurs when the torque is large or the unbalance is small. The second case occurs when the torque is small or the unbalance is large. If the torque and unbalance have intermediate values, then moderate growth in the cone angle will occur as the s/c passes through the PPL condition. The primary objective of this paper is to establish the relationships among the s/c inertia properties (including the size of the unbalance), the size of the despin torque, and the terminal cone angle at

despin.

II. Problem Formulation

Consider the idealized dual-spin s/c shown in Fig. 1. A despin torque motor applies a torque N to the rotor and an equal and opposite torque to the platform. The s/c is assumed to be free from external torques, so that its angular momentum is conserved and remains fixed in inertial space. Both of the s/c bodies are assumed to be completely rigid (no dissipation).

To obtain a set of governing equations, one can first formulate an expression for the angular momentum of the system using (1) - (4). Since no external forces act, one may differentiate this angular momentum and set the result equal to zero to obtain the following three equations:

$$I_1 \dot{\omega}_1 + I_{13}^B \dot{\omega}_B = \quad (5)$$

$$-\omega_2 (I_{33}^A \omega_A + I_{33}^B \omega_B - I_1 \omega_B) - I_{13}^B \omega_1 \omega_2$$

$$I_1 \dot{\omega}_2 = \quad (6)$$

$$\omega_1 (I_{33}^A \omega_A + I_{33}^B \omega_B - I_1 \omega_B) + I_{13}^B (\omega_1^2 - \omega_2^2)$$

$$I_{33}^A \dot{\omega}_A + I_{33}^B \dot{\omega}_B + I_{13}^B \dot{\omega}_1 = I_{13}^B \omega_2 \omega_B \quad (7)$$

$$\text{where } I_1 = I_{11}^A + I_{11}^B$$

A fourth equation is obtained by differentiating the angular momentum of A alone and setting the \hat{b}_3 component of the result equal to N , the despin torque. This yields

$$I_{33}^A \dot{\omega}_A = -N \quad (8)$$

Introduce the following scalings to obtain a set of dimensionless eqs. from (5) - (8):

$$\bar{\omega}_1 = \frac{\omega_1}{\omega_A(0)} \quad \bar{\omega}_2 = \frac{\omega_2}{\omega_A(0)}$$

$$\bar{\omega}_A = \frac{\omega_A}{\omega_A(0)} \quad \bar{\omega}_B = \frac{\omega_B}{\omega_A(0)}$$

$$\nu = \frac{I_{13}^B}{I_1} \quad \sigma = \frac{I_{33}^B}{I_1}$$

$$J = \frac{I_{33}^A}{I_{33}^B} \quad \tau = \omega_A(0) t$$

$$L = \frac{N}{\omega_A^2(0)} \left(\frac{1}{I_{33}^A} + \frac{1}{I_{33}^B} \right)$$

$$\lambda_o = \frac{I_{33}^A \omega_A + I_{33}^B \omega_B}{I_1 \omega_A(0)} = \sigma J \bar{\omega}_A + \bar{\omega}_B$$

$$i = \frac{d}{d\tau}$$

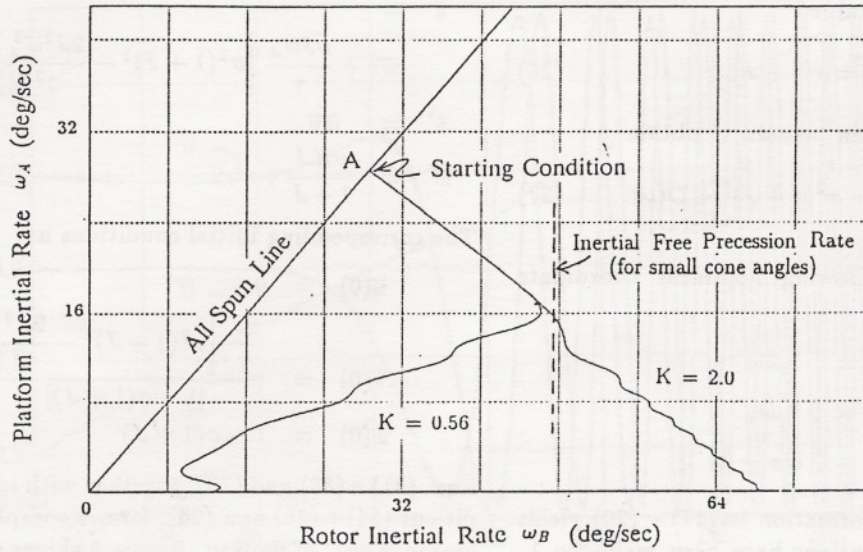


Figure 2: Despin Dynamics of an Unbalanced Dual-Spin S/C

The resulting eqs. are

$$\bar{\omega}'_1 + \nu \bar{\omega}'_B = \bar{\omega}_2(\bar{\omega}_B - \lambda_o) - \nu \bar{\omega}_1 \bar{\omega}_2 \quad (9)$$

$$\bar{\omega}'_2 = -\bar{\omega}_1(\bar{\omega}_B - \lambda_o) + \nu(\bar{\omega}_1^2 - \bar{\omega}_B^2) \quad (10)$$

$$\sigma \bar{\omega}'_B + \nu \bar{\omega}'_1 = \nu \bar{\omega}_2 \bar{\omega}_B + \frac{\sigma J L}{1 + J} \quad (11)$$

$$\bar{\omega}'_A = \frac{-L}{1 + J} \quad (12)$$

In what follows, we assume initial conditions which correspond to a state of steady spin about a principal axis when the despin motor torque, L , is zero and the relative rate between bodies A and B is also zero.

From (9) - (12), these conditions are

$$\bar{\omega}_1(0) = \frac{[1 - \sigma(1 + J)] + \sqrt{[1 - \sigma(1 + J)]^2 + 4\nu^2}}{2\nu} \quad (13)$$

$$\bar{\omega}_2(0) = 0 \quad (14)$$

$$\bar{\omega}_B(0) = 1 \quad (15)$$

$$\bar{\omega}_A(0) = 1 \quad (16)$$

III. Analytical Approach

In (9) - (16), ν , σ , J , and L are dimensionless parameters. L represents the relative size of the constant despin motor torque. σ and J represent the s/c mass properties. ν represents the size of the rotor dynamic unbalance. In what follows, we assume that ν is a small parameter ($\ll 1$). Since we

are interested in cases for which L is limited, we assume that $L = \nu K$, where K is a dimensionless parameter of $O(1)$. In the remainder of this derivation, terms of $O(\nu^2)$ are neglected. Under these assumptions, (9) - (16) may be simplified through algebraic manipulation and by truncating terms of $O(\nu^2)$. The simplified equations and initial conditions are

$$\bar{\omega}'_1 = \bar{\omega}_2(\bar{\omega}_B - \lambda_o) - \nu \bar{\omega}_1 \bar{\omega}_2 \quad (17)$$

$$\bar{\omega}'_2 = -\bar{\omega}_1(\bar{\omega}_B - \lambda_o) + \nu(\bar{\omega}_1^2 - \bar{\omega}_B^2) \quad (18)$$

$$\bar{\omega}'_B = \frac{\nu K J}{1 + J} + \frac{\nu \lambda_o \bar{\omega}_2}{\sigma} \quad (19)$$

$$\bar{\omega}'_A = \frac{-\nu K}{1 + J} \quad (20)$$

$$\bar{\omega}_1(0) = \frac{\nu}{[\sigma(1 + J) - 1]} \quad (21)$$

$$\bar{\omega}_2(0) = 0 \quad (22)$$

$$\bar{\omega}_B(0) = 1 \quad (23)$$

$$\bar{\omega}_A(0) = 1 \quad (24)$$

Now, derive an expression for the s/c angular momentum for its c.m. Define \bar{h} , a scaled momentum vector, as

$$\begin{aligned} \bar{h} &= \frac{\underline{h}}{\omega_A(0)I_1} \quad (25) \\ &= (\bar{\omega}_1 + \nu \bar{\omega}_B)\hat{b}_1 + (\bar{\omega}_2)\hat{b}_2 + (\lambda_o + \nu \bar{\omega}_1)\hat{b}_3 \end{aligned}$$

Using the initial conditions (21) - (24), the follow-

ing magnitude is obtained

$$\bar{h} = \sigma(1 + J) \quad (26)$$

Eqs. (25) and (26) can be used to obtain

$$\bar{\omega}_1^2 + \bar{\omega}_2^2 + \lambda_o^2 = \sigma^2(1 + J)^2 + O(\nu) \quad (27)$$

Introduce the following nonlinear coordinate transformation.

$$\begin{aligned} u &= \lambda_o \bar{\omega}_2 \\ v &= \lambda_o \bar{\omega}_1 \\ w &= \bar{\omega}_B - \lambda_o \end{aligned}$$

Applying this transformation to (17) - (20) yields (28) - (30). Substitutions have been made for λ_o from its definition above, and for $\bar{\omega}_1^2 + \bar{\omega}_2^2$ using (27).

$$u' = -wv + \nu F_1 \quad (28)$$

$$\begin{aligned} F_1 &= \frac{J\bar{\omega}_A}{\gamma} \left[\sigma^2(1 + J)^2 - \frac{2J^2\bar{\omega}_A^2}{\gamma^2} \right] \\ &+ \frac{w}{\gamma} \left[\sigma^2(1 + J)^2 - \frac{2J^2\bar{\omega}_A^2}{\gamma} \left(1 + \frac{3}{\gamma} \right) \right] \\ &- \frac{w^2 J \bar{\omega}_A}{\gamma} \left(1 + \frac{4}{\gamma} + \frac{6}{\gamma^2} \right) - \frac{w^3}{\gamma} \left(1 + \frac{2}{\gamma} + \frac{2}{\gamma^2} \right) \end{aligned}$$

$$v' = wu \quad (29)$$

$$w' = \frac{\nu K J}{1 + J} + \nu \gamma u \quad (30)$$

$$\text{where } \gamma = \frac{1}{\sigma} - 1$$

The F_1 term in (28) may be simplified through use of the method of averaging [5], here applied via the following near-identity transformation:

$$\begin{aligned} \bar{u} &= u \\ \bar{w} &= w \\ \bar{v} &= v - \nu G_1 \\ G_1 &= \frac{\sigma^2(1 + J)^2}{\gamma} - \frac{2J^2\bar{\omega}_A^2}{\gamma^2} \left(1 + \frac{3}{\gamma} \right) \\ &- \frac{wJ\bar{\omega}_A}{\gamma} \left(1 + \frac{4}{\gamma} + \frac{6}{\gamma^2} \right) \\ &- \frac{w^2}{\gamma} \left(1 + \frac{2}{\gamma} + \frac{2}{\gamma^2} \right) \end{aligned}$$

Eqs. (28) - (30) become (neglecting terms of $O(\nu^2)$ which arise when G_1 is differentiated)

$$\bar{u}' = \quad (31)$$

$$-\bar{w}v + \frac{\nu J \bar{\omega}_A}{\gamma} \left[\sigma^2(1 + J)^2 - \frac{2J^2\bar{\omega}_A^2}{\gamma^2} \right]$$

$$\bar{v}' = \bar{w}\bar{u} \quad (32)$$

$$\bar{w}' = \frac{\nu K J}{1 + J} + \nu \gamma \bar{u} \quad (33)$$

The corresponding initial conditions are

$$\bar{u}(0) = 0 \quad (34)$$

$$\bar{v}(0) = \frac{\nu J}{\gamma} \left[\sigma^2(1 + J)^2 - \frac{2J^2}{\gamma^2} \right] \quad (35)$$

$$\bar{w}(0) = 1 - \sigma(1 + J) \quad (36)$$

Eqs. (31) - (33) and (20), together with initial conditions (34) - (36) and (24), form a complete set of averaged eqs. of motion. Figure 3 shows computer simulation results generated using the exact eqs. (9) - (16) and the averaged eqs. In the figure, averaged results are overlayed on top of exact results. Excellent agreement is seen in this example.

PPL occurs when the inertial spin rate of the unbalanced body equals the free precession rate of the s/c, i.e., when $\bar{w} = 0$ in (31) - (33). We refer to the neighborhood of $\bar{w} = 0$ as the PPL resonance region. In this region, \bar{u} becomes increasingly negative. Growth in the magnitude of \bar{u} corresponds to growth in the cone angle.

The above set of averaged eqs. depend on four parameters. The next few steps will lead to a set of eqs. which depend only on two. First, rescale time and coordinates as follows:

$$t = \left(\sqrt{\frac{KJ}{1 + J}} \right) \tau$$

$$(\dot{\quad}) = \frac{d}{dt}$$

$$\bar{u} = \frac{KJx}{\gamma(1 + J)}$$

$$\bar{v} = \frac{KJy}{\gamma(1 + J)}$$

$$\bar{w} = \left(\sqrt{\frac{KJ}{1 + J}} \right) z$$

Eqs. (20) and (31) - (33) become

$$\dot{z} = -zy + \nu H_1 \quad (37)$$

$$H_1 = \frac{J\bar{\omega}_A \left(\sigma^2(1 + J)^2 - \frac{2J^2\bar{\omega}_A^2}{\gamma^2} \right)}{\left(\frac{KJ}{1 + J} \right)^{3/2}}$$

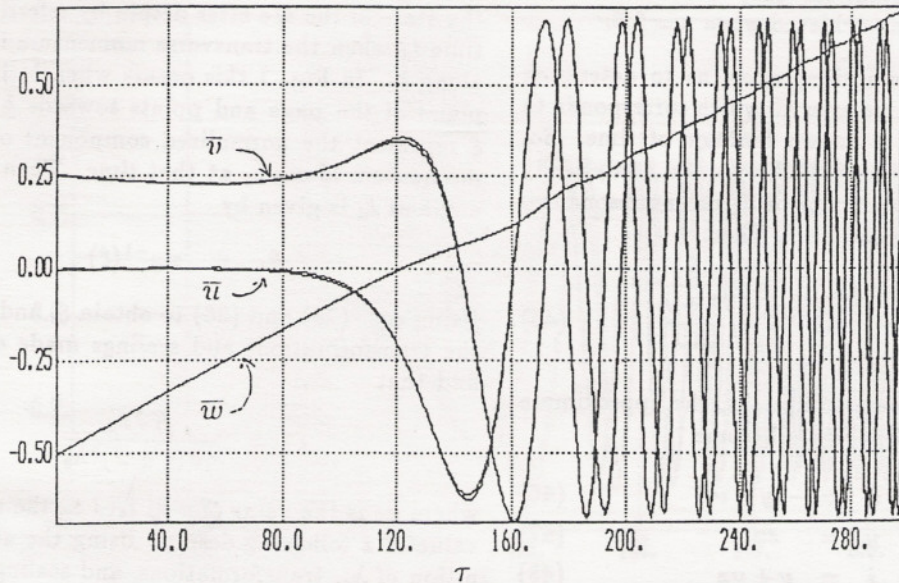


Figure 3: Comparison of Exact and Averaged Eqs. for Dual-Spin S/C
 $\nu = 0.005$ $\sigma = 0.666$ $J = 1.25$ $K = 1.5$

$$\dot{y} = zx \quad (38)$$

$$\dot{z} = \nu + \nu z \quad (39)$$

$$\dot{\bar{w}}_A = -\frac{\nu\sqrt{K}t}{\sqrt{J(1+J)}} \quad (40)$$

The corresponding initial conditions are

$$x(0) = 0 \quad (41)$$

$$y(0) = \frac{\nu(1+J)\left(\sigma^2(1+J)^2 - \frac{2J^2}{\gamma^2}\right)}{K[1-\sigma(1+J)]} \quad (42)$$

$$z(0) = \left(\sqrt{\frac{1+J}{KJ}}\right)[1-\sigma(1+J)] \quad (43)$$

$$\bar{w}_A(0) = 1 \quad (44)$$

Extensive computer simulations of (37) - (44) lead to the following observations:

1. The resonance region approximately extends from $z = -\sqrt{\nu}$ to $z = +\sqrt{\nu}$.
2. Almost all of the cone angle growth occurs while z is in this resonance region.
3. Prior to and following resonance, z grows linearly with time.
4. During resonance, z grows nearly linearly with time, for cases in which the final cone angle $< 30^\circ$.

5. Prior to resonance, y and z approximately satisfy the relationship $zy = H_1$, which implies that the s/c is in a state of quasi-steady spin about a principal axis, with small cone angle.

6. When $z \geq 0.8$ following resonance, the cone angle oscillates about its mean value with a nearly constant amplitude.

Note from (40) that \bar{w}_A is slowly time-varying, and therefore, H_1 is also slowly time-varying. Based on this fact and the above observations, we make the following key assumptions:

1. H_1 may be replaced by its average value over the resonance region.
2. For purposes of calculating the average value of H_1 :
 - (a) z may be considered to grow linearly with time.
 - (b) Resonance may be considered to begin when $z = -\sqrt{\nu}$ and end when $z = +\sqrt{\nu}$.
3. For purposes of determining the final cone angle:
 - (a) Initial conditions may be chosen corresponding to a state of steady spin about a principal axis, with a small cone angle and $z = -\sqrt{\nu}$.

- (b) Final conditions may be considered to have been achieved when $z = 0.8$.

The above assumptions allow us to determine an approximate time interval which corresponds to resonance. H_1 is a known function of time. So, having this time interval allows us to analytically calculate the average value of H_1 during resonance. Let this average value be β . Then

$$\beta = -\gamma \left[\sigma(1+J) \sqrt{\frac{1+J}{KJ}} \right]^3 \quad (45)$$

Under the above assumptions, the approximate eqns. of motion for the s/c become

$$\dot{z} = -zy + \nu\beta \quad (46)$$

$$\dot{y} = zx \quad (47)$$

$$\dot{z} = \nu + \nu z \quad (48)$$

$$z(0) = 0 \quad (49)$$

$$y(0) = -\sqrt{\nu}\beta \quad (50)$$

$$z(0) = -\sqrt{\nu} \quad (51)$$

Figure 4 depicts the variables x , y , and z generated from a computer simulation of the exact eqs. (9) - (12) using initial conditions (13) - (16), overlaid with the same variables generated from (46) - (51). Good agreement can be seen. Note that the initial conditions (49) - (51) used to generate approximate results correspond to the onset of resonance at $t \approx 105$ in the figure.

Eqs. (46) - (51) depend only on the two dimensionless parameters, ν and β , which are functions of the s/c mass properties and the magnitude of the despin motor torque. We wish to find an approximate expression for the mean value of the cone angle following despin, θ_d .

The cone angle can be calculated from the scaled angular momentum vector given in (25), once it has been normalized using the magnitude given in (26). To simplify this calculation, consider the nominal sequence of events as the despin maneuver is completed. When the platform rate ω_A reaches zero, the despin motor torque output drops to near zero. Thereafter, ω_A remains near zero. The rotor rate remains nonzero and is greater than the s/c precession rate. At this point, the platform has zero inertial angular momentum along \hat{b}_3 . Because of this and the axial symmetry of the platform, the s/c moves like a single rigid body. Thus, the components of transverse angular momentum along the \hat{b}_1 and \hat{b}_2 axes must oscillate about zero, 90° out of

phase with each other. We choose to characterize the state of the s/c after despin by referring to the time t_d when the transverse momentum is entirely along \hat{b}_2 . In Fig. 1 this occurs when \hat{b}_2 lies in the plane of the page and points towards \underline{h} . We let ξ represent the normalized component of angular momentum along \hat{b}_2 at that time. Then the cone angle at t_d is given by

$$\theta_d = \sin^{-1}(\xi) \quad (52)$$

Using eqs. (25) and (26) to obtain ξ , and applying the transformations and scalings made above, we find that

$$\xi = \frac{KJx_d}{\gamma\sigma(1+J)^2\lambda_o} \quad (53)$$

where x_d is the value of x at t_d , i.e., the maximum value of x following despin. Using the above definition of λ_o , transformations, and scalings we also have that

$$\lambda_o = \frac{1}{\gamma} \left(\sqrt{\frac{KJ}{1+J}} \right) z + \frac{J\bar{\omega}_A}{\gamma} \quad (54)$$

Following despin, λ_o simplifies since $\bar{\omega}_A$ is zero. Also, since z is observed numerically to increase nearly linearly throughout the despin process, we will assume that (48) can be replaced by $\dot{z} = \nu$ for the purpose of estimating θ_d . This gives

$$z_* = \nu t_* + z(0) \quad (55)$$

where t_* is the time for despin and $z(0)$ is given by eq. (43). In order to find t_* , we integrate (40) using initial condition (44). At the despin condition $\bar{\omega}_A = 0$, this gives

$$\bar{\omega}_A = 0 = \frac{-\nu\sqrt{K}t_*}{\sqrt{J(1+J)}} + 1$$

or

$$t_* = \frac{1}{\nu} \sqrt{\frac{J(1+J)}{K}} \quad (56)$$

Substituting from (56) into (55) gives

$$z_* = (1-\sigma)(1+J) \sqrt{\frac{1+J}{KJ}} \quad (57)$$

Substituting from (57) into (54) and assuming $\bar{\omega}_A = 0$ gives

$$\lambda_{o_*} = \frac{(1-\sigma)(1+J)}{\gamma} \quad (58)$$

Substituting from (58) into (53), and then substituting the result into (52) gives

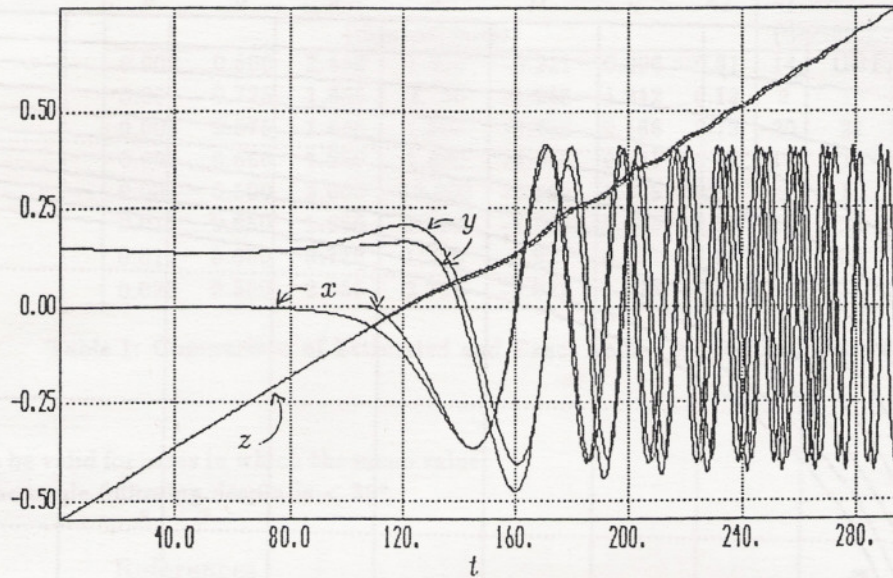


Figure 4: Comparison of Exact and Approx. Eqs. for Dual-Spin S/C
 $\nu = 0.005$ $\sigma = 0.666$ $J = 1.25$ $K = 1.5$

$$\theta_d = \sin^{-1} \left[\frac{KJx_d}{\sigma(1-\sigma)(1+J)^3} \right] \quad (59)$$

where x_d must be obtained from numerical integration of (46) - (51). We let η be the coefficient of x_d in (59). Then (59) may be rewritten as

$$\theta_d = \sin^{-1} (\eta x_d) \quad (60)$$

$$\eta = \frac{KJ}{\sigma(1-\sigma)(1+J)^3} \quad (61)$$

Eq. (60) is the desired expression for estimating the final cone angle. Because of the presence of the unbalance, there will be a small oscillation in the value of the cone angle following despin. Comparison with results from numerical integration of the exact eqs. (9) - (12) shows that (59) provides an estimate of the mean value of the cone angle following despin.

IV. Numerical Results

Using a computer simulation to numerically integrate eqs. (46) - (51), the quantity x_d may be generated for use in estimating the final cone angle via (60). A plot of x_d as a function of the two dimensionless parameters ν and β is shown in Figure 5. The plotted ranges of these two parameters correspond to the following ranges for the original four dimensionless parameters:

$$\begin{aligned} 0.002 &\leq \nu \leq 0.020 \\ 0.500 &\leq \sigma \leq 0.900 \\ 0.333 &\leq J \leq 3.000 \\ 1.300 &\leq K \leq 10.00 \end{aligned}$$

Note that for PPL to occur, we must have $\sigma < 1$ and $\sigma(1+J) > 1$. To avoid singularities in the coordinate scalings used in the derivation, we must also have $\sigma < 0.9$ and $\sigma(1+J) > 1.05$. The latter constraints must be satisfied when using the method developed in this paper.

The level curves on Figure 5 are for values of x_d equally spaced by an increment of 0.1, beginning with $x_d = 0.1$ and ending with $x_d = 1.0$. Points below the bottom curve correspond to cone angle estimates $\theta_d > 30^\circ$. Our method for obtaining estimates is only valid for $\theta_d < 30^\circ$.

As an example of how Fig. 5 may be used, consider a s/c whose mass properties and despin motor torque give the following values:

$$\begin{aligned} \nu &= 0.005 \\ \sigma &= 0.667 \\ J &= 1.250 \\ K &= 1.500 \end{aligned}$$

The value of β calculated for this case is -2.219 . The point at $(0.005, -2.219)$ (A) on Fig. 5 is close to the level curve $x_d = 0.400$. So we estimate the value of x_d to be 0.400. The value of η calculated for this case is 0.741. Substituting for x_d and η into

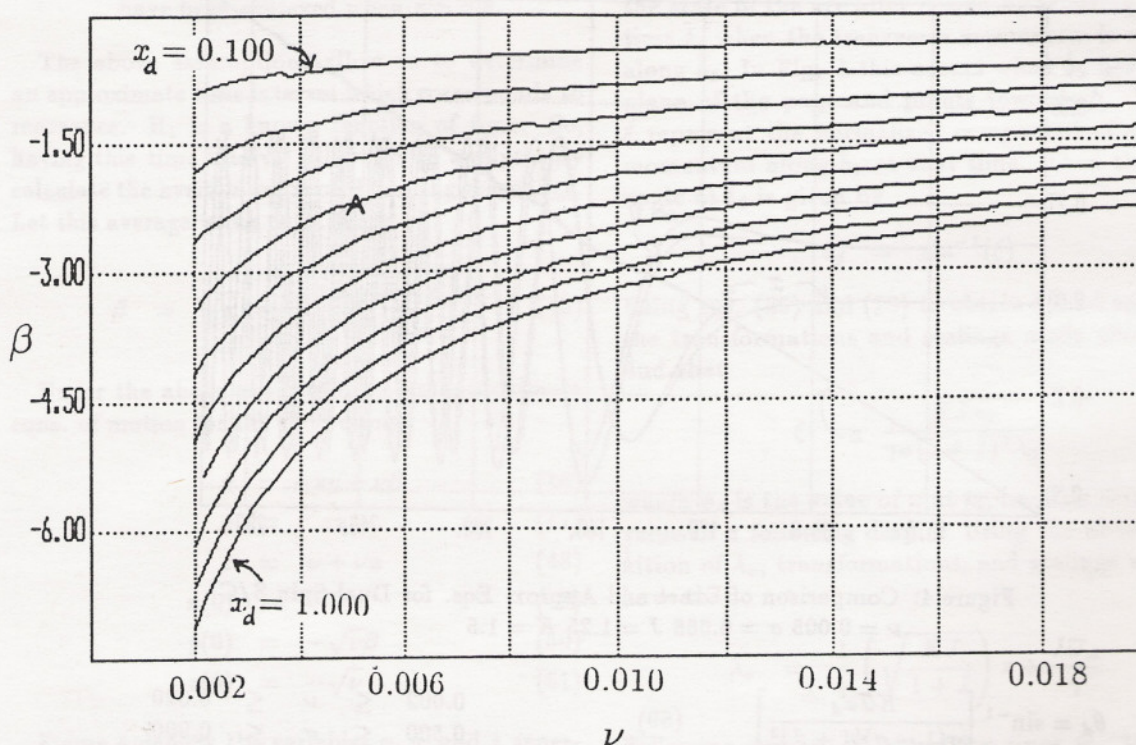


Figure 5: The Quantity x_d as a Function of ν and β

(60) gives an estimate for the cone angle following despin of 17° . The exact eqs. (9) - (12) were numerically integrated using initial conditions (13) - (16), for this same case. The numerically generated mean value of the cone angle following despin is about 18° . For this case, following despin the cone angle oscillates about its mean value with an amplitude of about 1° .

Table 1 summarizes several cases for which estimated and exact values of the mean of the final cone angle were obtained. The estimates were obtained using the method developed above. For the cases considered, the exact values θ_{EX} are generally within 10% of the estimates.

V. Summary

The dynamics of a dual-spin s/c undergoing despin through PPL has been investigated. A method has been developed for estimating the mean value of the cone angle following despin θ_d . From the four exact equations of motion, a set of three approximate eqs. are obtained. The approximate eqs. depend only on two dimensionless parameters, ν and β , which are functions of the s/c mass proper-

ties and the magnitude of the despin motor torque. The derivation of the approximate eqs. involves nonlinear coordinate transformations, the method of averaging, and several *ad hoc* assumptions. The estimate θ_d depends on the final value x_d of one of the states of the approximate eqs. Numerical integration of the approximate eqs. is used to generate plots of level curves of x_d versus ν and β . The method for estimating the mean value of the final cone angle consists of the following steps:

1. Calculate ν , β , and η from the s/c mass properties and the magnitude of the despin motor torque, using eqs. (45) and (61).
2. Use ν and β to locate a point on Figure 5.
3. Interpolate the value of x_d from the figure.
4. Use η and x_d in eq. (60) to estimate the cone angle.

Several cases have been presented which indicate that the method provides reasonably good estimates. The exact values of the mean of the final cone angle are generally within 10% of the estimated values, for the cases considered. Estimates

ν	σ	J	K	β	η	x_d	θ_d	θ_{EX}
(dimensionless)							(degrees)	
0.002	0.500	2.556	1.300	-6.221	0.296	0.81	14	15
0.002	0.725	1.666	2.750	-1.216	1.212	0.13	9	10
0.005	0.575	1.666	1.300	-3.638	0.468	0.73	20	21
0.005	0.666	1.250	1.500	-2.219	0.741	0.41	17	18
0.008	0.500	3.000	10.000	-0.389	1.875	0.08	9	10
0.011	0.650	1.666	2.750	-1.244	1.063	0.33	21	22
0.014	0.650	0.778	1.300	-1.937	0.791	0.63	30	28
0.020	0.500	2.556	2.750	-2.022	0.625	0.85	31	31

Table 1: Comparison of Estimated and Exact Values of the Final Cone Angle

appear to be valid for cases in which the mean value of the cone angle following despin is $< 30^\circ$.

References

- [1] M. P. Scher and R. L. Farrenkopf, "Dynamic Trap States of Dual-Spin Spacecraft", AIAA Journal, Vol. 12, No. 12, December 1974, pp. 1721-1725.
- [2] M. G. Hollars, "Minimum Energy Trap States of Dual-Spin Spacecraft", AIAA Guidance and Control Conference, August 1980, Collection of Technical Papers, Paper No. AIAA 80-1780, pp. 364-371.
- [3] J. E. Cochran, "Nonlinear Resonances in the Attitude Motion of Dual-Spin Spacecraft", J. Spacecraft, Vol. 14, No. 9, September 1977, pp. 562-572.
- [4] G. J. Adams, "Dual-Spin Spacecraft Dynamics During Platform Spinup", J. Guidance and Control, Vol. 3, No. 1, Jan.-Feb. 1980, pp. 29-36.
- [5] J. A. Sanders and F. Verhulst, *Averaging Methods in Nonlinear Dynamical Systems*, Applied Mathematical Sciences, Vol. 59, Springer-Verlag, 1985.