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Corrections to Chapter 3 on Averaging in "Topics in Nonlinear Dynamics with Computer Algebra".

<u>The main idea</u>: Throughout Chapter 3, the generating functions w_i, v_i and u_i involve arbitrary functions of $\bar{a}$, referred to as $K_i(\bar{a})$, see e.g., eqs.(3.4.3) and (3.4.4) on p.100. The treatment of these quantities is incorrect. The error is that the derivatives of the $K_i(\bar{a})$ are omitted throughout the Chapter. E.g. eq.(3.4.5) reads

$$\dot{\bar{a}} = -\epsilon\frac{\bar{a}}{2} + \frac{1}{2}\epsilon^2 K_1(\bar{a}) + O(\epsilon^3) \qquad (3.4.5)$$

$$\dot{\bar{\varphi}} = 1 - \frac{1}{8}\epsilon^2 + O(\epsilon^3)$$

whereas it should read:

$$\dot{\bar{a}} = -\epsilon\frac{\bar{a}}{2} - \frac{1}{2}\epsilon^2 K_1(\bar{a}) + \frac{1}{2}\epsilon^2\bar{a}\frac{dK_1}{d\bar{a}} + O(\epsilon^3) \qquad (3.4.5)$$

$$\dot{\bar{\varphi}} = 1 - \frac{1}{8}\epsilon^2 + \frac{1}{2}\epsilon^2\bar{a}\frac{dK_2}{d\bar{a}} + O(\epsilon^3)$$

<u>Corrections</u>: The corrected equations used in second and third order averaging in this Chapter may be obtained from the MACSYMA programs in Appendices 6 and 7 by adding the line

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depends([k1,k2,k3,k4],abar);
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at the beginning of both programs.

Many of the equations given in Chapter 3 are nevertheless correct, due to the fortuitous fact that the functions K_i were generally chosen as constants, i.e. independent of $\bar{a}$, see e.g. p.101. In these cases, the missing terms which involve the derivatives of K_i with respect to $\bar{a}$ happen to be zero. However, the treatment of van der Pol's eq. in section 3.5 is incorrect, since in eq.(3.5.7) the function K_3 is selected to be a specific function of $\bar{a}$. The treatment of van der Pol's eq. may be corrected by replacing the text starting on the bottom of p.105 and continuing on p.106 by the following:

If we let K_1, K_2, K_3 and K_4 respectively represent the arbitrary functions of $\bar{a}$ which are associated with w_1, w_2, v_1 and v_2 (cf. eqs.(3.3.3),(3.3.5),(3.4.3) and (3.4.4)), then the program in Appendix 7 results in the following averaged eqs.:

(3.5.5)
$$\dot{\bar{a}} = \epsilon \frac{\bar{a}}{8}(4-\bar{a}^2) + \epsilon^2 \frac{K_1}{8}(4-3\bar{a}^2) + \epsilon^2 \frac{\bar{a}}{8}(\bar{a}^2-4)\frac{dK_1}{d\bar{a}}$$
$$+ \epsilon^3 \left[-\frac{9}{256}\bar{a}^3 + \frac{39}{2048}\bar{a}^5 - \frac{43}{16384}\bar{a}^7 + K_3\left[\frac{1}{2}-\frac{3}{8}\bar{a}^2\right] - \frac{3}{8}K_1\bar{a}\right]$$
$$+ \epsilon^3 \left[K_1\frac{dK_1}{d\bar{a}}\left[\frac{3}{8}\bar{a}^2-\frac{1}{2}\right] + \left[\left[\frac{dK_1}{d\bar{a}}\right]^2 - \frac{dK_3}{d\bar{a}}\right]\frac{\bar{a}}{8}(4-\bar{a}^2)\right] + O(\epsilon^4)$$

(3.5.6)
$$\dot{\bar{\varphi}} = 1 + \epsilon^2\left[-\frac{1}{8}+\frac{3}{16}\bar{a}^2 - \frac{11}{256}\bar{a}^4\right] + \epsilon^2\frac{\bar{a}}{8}(\bar{a}^2-4)\frac{dK_2}{d\bar{a}}$$
$$+ \epsilon^3 K_1 \bar{a}\left[\frac{3}{8}-\frac{11}{64}\bar{a}^2\right]$$
$$+ \epsilon^3\left[K_1\frac{dK_2}{d\bar{a}}\left[\frac{3}{8}\bar{a}^2-\frac{1}{2}\right] + \left[\frac{dK_1}{d\bar{a}}\frac{dK_2}{d\bar{a}} - \frac{dK_4}{d\bar{a}}\right]\frac{\bar{a}}{8}(4-\bar{a}^2)\right] + O(\epsilon^4)$$

As expected, the form of the averaged equations depends upon the choice of the arbitrary functions $K_i(\bar{a})$. E.g., a judicious choice of the K_i can make these equations easier to handle: if we take $K_1 = K_2 = K_4 = 0$ and

(3.5.7)
$$K_3 = \frac{\bar{a}}{128}(\bar{a}^2-4)\ln(\bar{a}^2-4) + \frac{43}{4096}\bar{a}^3(\bar{a}^2-4) - \frac{\bar{a}}{256}$$

then the averaged eqs.(3.5.5) and (3.5.6) become:

(3.5.8)
$$\dot{\bar{a}} = \epsilon\frac{\bar{a}}{8}(4-\bar{a}^2) + O(\epsilon^4), \quad \dot{\bar{\varphi}} = 1 + \epsilon^2\left[-\frac{1}{8}+\frac{3}{16}\bar{a}^2-\frac{11}{256}\bar{a}^4\right] + O(\epsilon^4)$$

This completes the corrections on p.106. The rest of the Chapter appears to be free of this particular error.