Dynamics of Fruit Tree Trunk Impact

S. K. Upadhyaya, R. H. Rand, J. R. Cooke
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ABSTRACT

THE theory of colliding bodies is applied to the problem of impact harvesting of tree fruits. We found for the impaction of a pendulum or a spring loaded mass with a tree trunk:

1 that a simple one degree of freedom model is an adequate representation of the impact response of the tree,

2 that multiple impacts are shown to exist, in agreement with experimental studies,

3 that certain physical properties of a tree relevant to the design of impact harvesting equipment (e.g., dynamic mass, damping coefficient, spring stiffness and Young's modulus) can be obtained from a pendulum test, and

4 that the energy transferred from the pendulum to the tree is not simply the difference between the initial and final potential energy of the pendulum or the spring loaded mass when at zero velocity. The error due to neglecting the loss in mechanical energy may be as high as 40 to 50 percent, which is a significant portion of the energy input. Therefore, it is necessary to take into account the mechanical energy lost due to impact in such studies.

INTRODUCTION AND REVIEW OF LITERATURE

The desire to reduce impact damage to seeds, fruits and vegetables during harvest and handling has led to investigations of their impact behavior. The information obtained from impact testing has been used to determine the physical properties of agricultural materials. Several different impact testing methods have been used. Drop tests, which consist of dropping the material to be tested onto a hard surface or dropping a plunger onto the material or dropping one fruit onto the other have been widely used (Angle and O'Brien, 1974; Clark, 1971; Davis and Rehkugler, 1971; Fletcher, 1971; Fluck and Ahmed, 1973; Hammerle and Mohsenin, 1966; Hoag, 1972; Mohsenin and Göehlich, 1962; Voisey and Hunt, 1967; Wright and Splinter, 1968). The materials tested were mainly fruits but also included soybean pods (Hoag, 1972), sweet potatoes (Wright and Splinter, 1968) and egg shells (Voisey and Hunt, 1967). A pendulum device has also frequently been used (Bilanski, 1966; Bittner et al., 1967; Hoag, 1972; Horsfield et al., 1972; Jindal and Mohsenin, 1976, 1978; Nelson and Mohsenin, 1968; Parke, 1963; Prasad and Gupta, 1975; Moore et al., 1971; Splinter et al., 1962; Zoerb and Hall, 1960). Moore et al. (1979); Prasad and Gupta (1975) and Splinter et al. (1962) used a knife mounted on a pendulum to impact a plant in order to cut the stalk or detach the leaves. Seeds were given high velocity impacts using a centrifugal gun (Cooke and Dickens, 1971) or a rotating paddle, arm or flywheel (Bilanski, 1966; Burkhardt and Stout, 1971; Clark et al., 1969). Fridley and Adrian (1966b) used a plunger mounted on a spring loaded arm to impact peaches, pears, apricots and apples in order to study their mechanical properties. Mitchell and Rounthwaite (1964) used a rotating hammer to impact wheat seeds. Perry and Hall (1966) used a spring loaded striking bar to impact pea beans. A rotating drum was used by Turner et al. (1967) to impact peanuts. Pneumatic devices were also used to impact seeds. Seeds were accelerated pneumatically and impacted onto a flat surface (Keller et al., 1972; Kirk and McLeod, 1967). Fletcher et al. (1965) used an air speed regulated piston to obtain high speed loading for fruits and vegetables.

The cushioning ability (energy absorbing capacity) of padding materials used in catching surfaces has been investigated using an impact technique (Fridley et al., 1964; Mohsenin et al., 1978; Simpson and Rehkugler, 1972).

Several attempts were made to theoretically investigate the impact behavior of agricultural products. Arnold and Roberts (1966) gave the Hertz solution for the contact stress between wheat seeds. Cooke and Dickens (1971) analyzed a centrifugal gun for impact testing of seeds. Visco-elastic models have been used by several researchers to study the impact behavior of fruits (Franke and Rohrbach, 1979; Hamann, 1970; Kennish and Henderson, 1978; Rumsey and Fridley, 1977).

An important problem that has received very little attention in agricultural engineering literature is that of the impact of a moving mass against an elastic member. This problem is of considerable interest, especially in tree-fruit impact harvesting introduced by Fridley and Adrian (1969). Tree-trunk impact or tree-limb impact harvesting is believed to have the potential of reducing detachment and predetachment damage to fruit as it avoids excessive motion prior to or at the time of detachment. Pellerin et al. (1978a) developed a pendulum impact tree trunk harvester; Pacheco (1978) developed a spring loaded billet type tree trunk shaker. Experimental results obtained with these machines in the field support the above belief (Pellerin et al., 1978a, b). A finite element study of the dynamics of limb impact harvesting revealed that the fruit moves very little, if any, when the base of the limb is impacted (Upadhyaya, 1979; Upadhyaya et al., 1979). This model wa verified (Upadhyaya, 1979; Upadhyaya et al., 1979).

Pellerin et al. (1978a) observed that when a heavy pendulum struck the tree trunk the pendulum moved with the tree for a short duration after the impact. Srivastava

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The authors are: S. K. UPADHYAYA, Research Associate, Agricultural Engineering Dept., R. H. RAND, Associate Professor, Theo. and Appl. Mech. Dept., and J. R. COOKE, Professor Agricultural Engineering Dept., Cornell University, Ithaca, NY.

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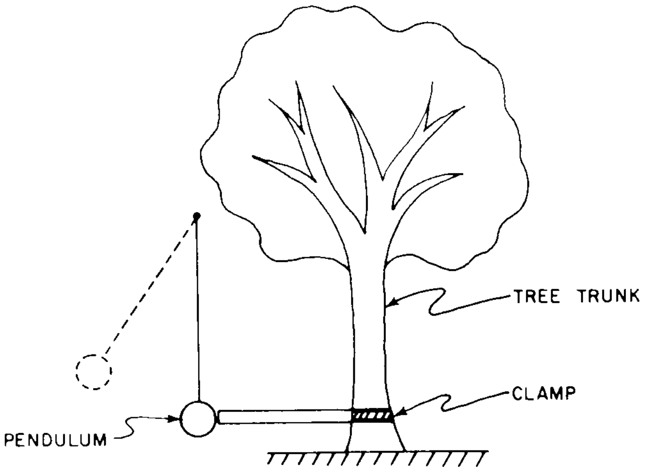

FIG. 1 The schematic diagram of a pendulum impacting a tree.

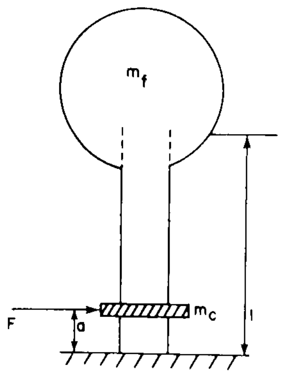

FIG. 2 A single degree of freedom model for the tree impact problem

and Rehkugler (1976) observed a similar phenomenon when a pendulum impacted a tractor frame. Scura (1976) found a similar behavior when a pendulum impacted a soil column. A close examination of the phenomena by Pellerin et al. (1978a) revealed that the pendulum does not stay in contact with the object immediately after the impact, but rather collides and separates repeatedly until the pendulum and the object move together with a common velocity. After moving with a common velocity for a while, the pendulum and the object eventually separate.

This study investigates the dynamics of such an impact and presents inferences that can be made about the properties of the object impacted. Furthermore, this study also estimates the mechanical energy dissipated due to impact.

THEORETICAL DEVELOPMENT

Goldsmith (1960) has shown that the response of a beam impacted by a falling mass can be approximated by a spring-mass model. In this analysis we will show that the deflection of the point of impact of a tree, impacted with a pendulum or a heavy mass can be described by a spring-mass-damper (one degree of freedom) system.

One Degree of Freedom Model

Let us model an apple tree as a beam of length ℓ, with a uniform cross sectional area A. We assume that all of the fruit-bearing branches can be treated as equivalent to a lumped mass at one end of this trunk and that the other end is rigidly fixed to the ground. Let m_f be the total mass of all the fruits, branches and leaves at the far end of the trunk at a distance ℓ above the ground (Figs. 1 and 2). Let m_c be the mass of the clamp where the impact force, F, is applied. Let a be the distance from the base of the tree to the point of impact loading. Furthermore, it is assumed that the damping force is proportional to velocity (Upadhyaya, 1979a).

Under the assumption of small deflections and small changes in slope of the trunk, we can apply the Bernoulli-Euler theory (Upadhyaya, 1979a) and obtain:

$$\hat{y} = \frac{F\hat{x}^2}{6EI}(3a - \hat{x}) \qquad 0 \leqslant \hat{x} \leqslant a$$

$$= \frac{Fa^2}{6EI}(3\hat{x} - a) \qquad a \leqslant \hat{x} \leqslant \ell \qquad [1]$$

where,

$\hat{y}$ = static displacement of the beam, m
$\hat{x}$ = the distance above the ground level where deflection $\hat{y}$ is measured
E = Young's modulus of elasticity, N/m²
I = area moment of inertia about the neutral axis of the beam, m⁴

In deriving the above relations the transverse shear in the beam and the weight of the beam are neglected.

Let $w(\hat{x},t)$ be the transverse displacement of the beam under the action of an impact force. Following Goldsmith (1960) let us assume a solution of the form:

$$w(\hat{x},t) = y(\hat{x})f(t) \qquad [2]$$

Therefore,

$$w(a,t) = y(a)f(t) \qquad [3]$$

From equations [2] and [3] we obtain,

$$w(\hat{x},t) = \frac{y(\hat{x})}{y(a)} w(a,t) \qquad [4]$$

Assuming $w(a,t) = \eta(t)$ and $y(\hat{x})$ equal to the static mode shape (Goldsmith, 1960), we get,

$$w(\hat{x},t) = \frac{\hat{x}^2(3a-\hat{x})}{2a^3}\eta(t) \qquad 0 \leqslant \hat{x} \leqslant a$$

$$= \frac{(3\hat{x}-a)}{2a}\eta(t) \qquad a \leqslant \hat{x} \leqslant \ell \qquad [5]$$

Lagrange's equation of motion for this case may be written as (Greenwood, 1965, p. 229-280).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) + \frac{\partial P}{\partial \dot{\eta}} - \frac{\partial L}{\partial \eta} = Q \qquad [6]$$

where,

L = T-V = Lagrangian, N·m
P = Rayleigh dissipation function, N·m
Q = Generalized force, N·m
T = Kinetic energy, N·m
V = Potential energy, N·m

The kinetic energy T is given by,

$$T = \int_0^\ell \frac{1}{2}\rho A\,\dot{w}(\hat{x},t)^2\,d\hat{x} + \frac{1}{2}m_c\,\dot{w}(a,t)^2 + \frac{1}{2}m_f\,\dot{w}(\ell,t)^2$$

where · denotes differentiation with respect to time and ρ

is the density of the tree trunk material.

$$T = \frac{1}{2} \rho A \left\{ \int_0^a \left[\frac{\hat{x}^2 (3a-\hat{x})}{2a^3} \right]^2 d\hat{x} + \int_a^\ell \left[\frac{(3\hat{x}-a)}{2a} \right]^2 d\hat{x} \right\} \dot{\eta}^2$$

$$+ \frac{1}{2} m_c \dot{\eta}^2 + \frac{1}{2} m_f \left(\frac{3\ell-a}{2a}\right)^2 \dot{\eta}^2$$

$$T = \frac{M_T}{2} \dot{\eta}^2 \qquad [7]$$

where

$$M_T = m_t \left[\frac{3\ell(\ell-a) + a^2}{4a^2} - \left(\frac{a}{70\ell}\right) \right] + \left[m_c + m_f \left(\frac{3\ell-a}{2a}\right)^2 \right]$$

$m_t = \rho A \ell$ = mass of the tree trunk, kg

The potential energy V is given by,

$$V = \frac{EI}{2} \int_0^\ell \left(\frac{\partial^2 w}{\partial \hat{x}}\right)^2 d\hat{x} = \frac{EI}{2} \int_0^a \left(\frac{\partial^2 w}{\partial \hat{x}^2}\right)^2 d\hat{x}$$

$$= \frac{EI}{2} \int_0^a \left[\frac{3(a-\hat{x})}{a^3} \right]^2 d\hat{x}\, \eta^2$$

$$= \frac{K_T}{2} \eta^2 \qquad [8]$$

where

$$K_T = 3EI/a^3$$

The Rayleigh dissipation function P is given by

$$P = \frac{C}{2} \int_0^\ell \left(\frac{\partial w}{\partial t}\right)^2 d\hat{x} \qquad [9]$$

$$P = C/2 \left\{ \int_0^a \left[\frac{\hat{x}^2 (3a-\hat{x})}{2a^3} \right]^2 d\hat{x} + \int_a^\ell \left[\frac{3\hat{x}-a}{2a} \right]^2 d\hat{x} \right\} \dot{\eta}^2$$

$$= \frac{C_T \dot{\eta}^2}{2} \qquad [10]$$

where

$$C_T = \frac{C\ell}{2} \left[\frac{3\ell(\ell-a) + a^2}{4a^2} - (1/70)(a/\ell) \right]$$

From a consideration of the principle of virtual work, Q can be shown to be equal to the applied external impact force, F (Greenwood, 1965, p. 229-280).

From equations [6] to [10] the equations of motion of the tree may be obtained as:

$$M_T \ddot{\eta} + C_T \dot{\eta} + K_T \eta = F \qquad [11]$$

Note that the presence of the taper along the trunk would not change the nature of the problem, although it would change the expressions for M_T, C_T and K_T. By following the above derivation an equation similar to equation [11] can be obtained for a tapered trunk. The corresponding calculations, however, are of greater algebraic complexity and are not necessary in problems with a very small taper, as is the case here.

Equation [11] indicates that under the action of an external force, the tree can be regarded as a single degree of freedom system. Note that the dynamic mass of the tree, M_T, is different than its static mass. In this analysis, we will assume that equation [11] describes the conditions of impact fruit harvesting.

Multiple Impacts:

Goldsmith (1960) assumed an inelastic impact when a steel mass impacted a steel beam transversely. This means that there exists a single impact and that the two bodies move together after the impact. However, Pellerin et al. (1978b) experimentally found more than one impact. This implies that the coefficient of restitution, e, is not zero.

Consider the system shown in Fig. 1. Following the first impact with the tree, the pendulum continues to move towards the tree. Although the point of impact on the tree receives a high velocity upon this first impact, it is restrained by the high stiffness and damping of the tree. Therefore, it slows rapidly. As a result the pendulum meets the tree once again, leading to a second impact. This process repeats until the relative velocity of the tree and the pendulum is zero. Then the two bodies move together until the contact force between them becomes zero, at which time they separate again.

Since the pendulum has been observed to move only slightly to the right (Fig. 1) following the first impact (Pellerin et al., 1978, and Srivastava unpublished experimental data), we will use a small angle approximation to simplify the differential equation of motion of the pendulum. Assuming a small rotation for the pendulum after the first impact and until the separation occurs, and using equation [11], we can represent the impact process by the simplified model shown in Fig. 3. Here the pendulum of Fig. 1 is replaced by a concentrated mass, m and a linear spring, k.

We will analyze the dynamics of the above impact process for the special case in which the pendulum continues to move towards the tree during the whole process (i.e., y(t) increases monotonically). The experimental results show that this is the case during impact fruit harvesting. Let us model the above process with a possibly infinite sequence of impacts. Following the i^{th} impact let the two masses separate with velocities V_i (tree) and v_i (pendulum). Let their approach velocities for the $(i+1)^{th}$ impact be U_{i+1} (tree) and u_{i+1} (pendulum). Let us proceed to establish a relation between the above i^{th} and the $(i+1)^{th}$ impact velocities. Note that the two bodies are moving freely between the i^{th} and the $(i+1)^{th}$ impact. For the $(i+1)^{th}$ impact to occur the following inequalities must hold:

$$V_i > v_i$$

and $\qquad [12]$

$$u_{i+1} > U_{i+1}$$

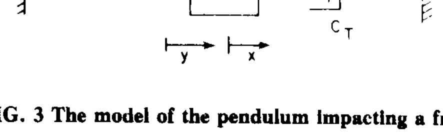

FIG. 3 The model of the pendulum impacting a fruit tree.

During the interval between the i^{th} and $(i+1)^{th}$ impacts the equations of motions for these two bodies are:

$$M_T \ddot{x} + C_T \dot{x} + K_T x = 0 \quad \text{with} \begin{cases} x = x^\circ_i \\ \dot{x} = V_i \end{cases} \text{at } t = \tau_i$$

$$m\ddot{y} + \frac{mg}{\ell_e} y = 0 \quad \text{with} \begin{cases} y = y^\circ_i \\ \dot{y} = v_i \end{cases} \text{at } t = \tau_i \qquad [13]$$

where

ℓ_e = equivalent length of a simple pendulum, m

τ_i = instant of i^{th} impact, s

x,y = displacement of the tree and the pendulum, respectively, m

x_i°, y_i° = displacements of the tree and the pendulum, respectively at time $t = \tau_i$, s.

The above equations may be written as:

$$\ddot{x} = -\omega_1^2 x - 2\xi \omega_1 \dot{x}$$
$$\ddot{y} = -\omega_2^2 y \qquad [14]$$

where

$$\omega_1{}^2 = K_T/M_T$$
$$2\xi\omega_1 = C_T/M_T$$
$$\omega_2{}^2 = g/\ell_e$$

Integrating between τ_i and τ_{i+1}, we get

$$\dot{x} \Big|_{\tau i}^{\tau_{i+1}} = -\omega_1^2 \int_{\tau_i}^{\tau_{i+1}} x dt - 2\xi\omega_1 \, x \Big|_{\tau i}^{\tau_{i+1}},$$

and

$$\dot{y} \Big|_{\tau_i}^{\tau_{i+1}} = -\omega_2{}^2 \int_{\tau_i}^{\tau_{i+1}} y dt .$$

Therefore,

$$V_i - U_{i+1} = \omega_1{}^2 A_x^i + 2\xi\omega_1 \Delta x^i = R \qquad [15]$$

and

$$v_i - u_{i+1} = \omega_2{}^2 A_y^i \qquad [16]$$

where

$$A_x^i = \int_{\tau_i}^{\tau_{i+1}} x dt$$
$$A_y^i = \int_{\tau_i}^{\tau_{i+1}} y dt$$
$$\Delta x^i = x_{i+1} - x_i .$$

Note that the quantities A_x^i and A_y^i represent the area under the respective displacement-time plots. Subtracting equation [16] from equation [15] we get,

$$V_i - v_i - (U_{i+1} - u_{i+1}) = R - \omega_2{}^2 A_y^i . \qquad [17]$$

If we define

$$\gamma_i = V_i - v_i \qquad [18]$$

then equation [12] gives

$$\gamma_i > 0 \qquad [19]$$

In addition to the above relationships which represent the motion of the tree and pendulum system between any two consecutive impacts, we have the following relations which characterize any given impact:

(a) Conservation of momentum during an impact

(b) Loss of mechanical energy during an impact

The conservation of momentum during the i^{th} impact requires that

$$M_T U_i + mu_i = M_T V_i + m v_i \qquad [20]$$

The loss of mechanical energy during the i^{th} impact is determined with the help of an equation which defines the coefficient of restitution, e:

$$e = -\frac{(V_i - v_i)}{(U_i - u_i)} \qquad [21]$$

The coefficient of restitution e, depends on the characteristics of impacting bodies. Although it depends on the shape, size and impact velocities of the bodies to some extent, we neglect these effects in this analysis. Using equations [15] to [21], we can establish a relation between the separation velocities of the i^{th} and $(i+1)^{th}$ impacts as follows. From equations [21], [17] and [18] we get,

$$\gamma_i + \frac{\gamma_{i+1}}{e} = R - \omega_2{}^2 A_y^i \qquad [22]$$

Multiplying equation [15] by M_T and equation [16] by m and adding:

$$M_T(V_i - U_{i+1}) + m(v_i - u_{i+1}) = M_T R + m\omega_2{}^2 A_y^i$$

or

$$M_T V_i + mv_i - (M_T U_{i+1} + mu_{i+1}) = M_T R + m\,\omega_2{}^2 A_y^i . \qquad [23]$$

Substituting equations [18] and [20] into equation [23], we get

$$M_T(v_i + \gamma_i) + mv_i - [M_T(v_{i+1} + \gamma_{i+1}) + mv_{i+1}] = M_T R + m\,\omega_2{}^2 A_y^i$$

or

$$(M_T + m)(v_i - v_{i+1}) = M_T(\gamma_{i+1} - \gamma_i) + M_T R + m\,\omega_2{}^2 A_y^i \ldots \qquad [24]$$

Substituting for R from equation [22] in equation [25], we obtain upon simplification

$$v_i - v_{i+1} = \left(\frac{M_T}{M_T + m}\right)\left(\frac{1+e}{e}\right)\gamma_{i+1} + \omega_2{}^2 A_y^i . \qquad [25]$$

Since we assumed that y(t) is monotonically increasing (i.e., the pendulum moves in the same direction), the velocity v_i and the area under the displacement vs. time plot A_y^i are positive. From equation [19], γ_{i+1} is positive. Therefore, the right hand side of equation [25] is positive, which gives

$$v_i > v_{i+1} > 0 \qquad [26]$$

or

$$\frac{v_{i+1}}{v_i} < 1 .$$

However, it is possible that the right hand side of equation [25] vanishes as i approaches ∞. In this case, v_{i+1} approaches v_i as i approaches ∞.
Thefore,

$$\lim_{i\to\infty} \frac{v_{i+1}}{v_i} \leqslant 1. \qquad [27]$$

Thus, two distinct possibilities occur,

$$\text{case (i)} \lim_{i\to\infty} \frac{v_{i+1}}{v_i} < 1 \qquad [28]$$

$$\text{case (ii)} \lim_{i\to\infty} \frac{v_{i+1}}{v_i} = 1. \qquad [29]$$

Consider case (i):

$$\lim_{i\to\infty} \frac{v_{i+1}}{v_i} < 1.$$

Equation [28] means that

$$\sum_{i=1}^{\infty} v_i$$

converges by the ratio test (Sokolnikoff and Redheffer, 1966, p. 18). Therefore,

$$\sum_{i=1}^{\infty} v_{i+1}$$

also converges. This implies that

$$\sum_{i=1}^{\infty} (v_i - V_{i+1})$$

converges.
From equation [25], as $A_y^i > 0$

$$v_i - v_{i+1} > \left(\frac{M_T}{M_T+m}\right)\left(\frac{1+e}{e}\right) \gamma_{i+1}$$

or

$$\sum_{i=1}^{\infty} (v_i - v_{i+1}) > \left(\frac{M_T}{M_T+m}\right)\left(\frac{1+e}{e}\right) \sum_{i=1}^{\infty} \gamma_{i+1}. \qquad [30]$$

Since

$$\sum_{i=1}^{\infty} (v_i - v_{i+1})$$

converges,

$$\sum_{i=1}^{\infty} \gamma_{i+1}$$

also converges by the comparison test (Sokonikoff and Redheffer, 1966, p. 15-17). Therefore,

$$\sum_{i=1}^{\infty} \gamma_i$$

converges.

$$\text{This means that} \lim_{i\to\infty} \gamma_i = 0. \qquad [31]$$

Therefore, from equation [18]

$$\lim_{i\to\infty} v_i = \lim_{i\to\infty} V_i$$

and the two bodies move together.
Now consider case (ii)

$$\lim_{i\to\infty} \frac{v_{i+1}}{v_i} = 1.$$

Let us assume that

$$\lim_{i\to\infty} v_i = \lim_{i\to\infty} v_{i+1} = V.$$

Then from equation [30], for N impacts, we have,

$$\sum_{i=1}^{N} (v_i - v_{i+1}) > \left(\frac{M_T}{M_T+m}\right)\left(\frac{1+e}{e}\right) \sum_{i=1}^{N} \gamma_{i+1}$$

or

$$v_1 - v_{N+1} > \left(\frac{M_T}{M_T+m}\right)\left(\frac{1+e}{e}\right) \sum_{i=1}^{N} \gamma_{i+1} .$$

Since

$$\lim_{N\to\infty} v_{N+1} = V,$$

from the above equation we get,

$$v_1 - V > \left(\frac{M_T}{M_T+m}\right)\left(\frac{1+e}{e}\right) \sum_{i=1}^{\infty} \gamma_{i+1} . \qquad [32]$$

Thus

$$\sum_{i=1}^{\infty} \gamma_{i+1}$$

has positive terms, (equation [19]) and is bounded (equation [32]). Therefore, this series converges by the principle of monotone convergence (Sokolnikoff and Redheffer, 1966, p. 9). This implies that

$$\sum_{i=1}^{\infty} \gamma_i$$

converges.
This in turn means that

$$\lim_{i\to\infty} \gamma_i = 0$$

or

$$\lim_{i\to\infty} v_i = \lim_{i\to\infty} V_i = V \text{ (from equation [18])}.$$

Thus, the two bodies move together.
In the context of tree fruit harvesting, case (i) appears to be a special case because equation [31] implies that

$$\lim_{i\to\infty} v_i = \lim_{i\to\infty} V_i$$

But as discussed earlier equation [28] proves the convergence of

$$\sum_{i=1}^{\infty} v_i$$

This convergenee implies that

$$\lim_{i\to\infty} v_i = 0$$

Therefore, in case (i), the two bodies come to rest as a result of the impact process. Experimental results suggest that case(ii) is more common (Pellerin et al., 1978b). Note that the conclusion of case (i) can be derived as a special case of case (ii) when

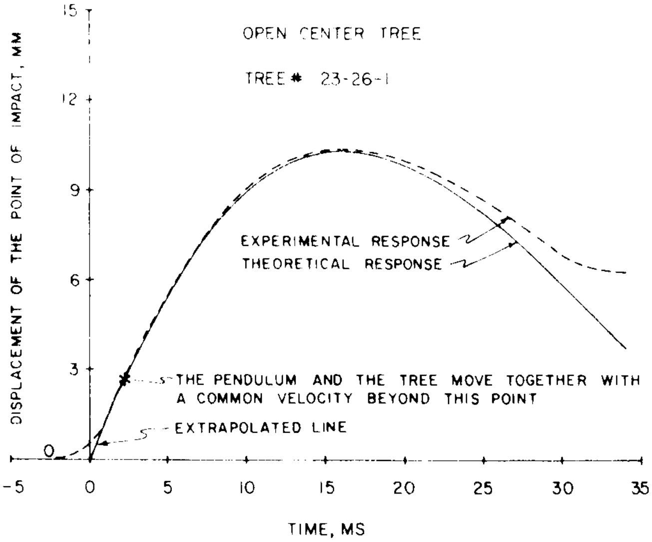

FIG. 4 Experimental and theoretical response of an open center apple tree.

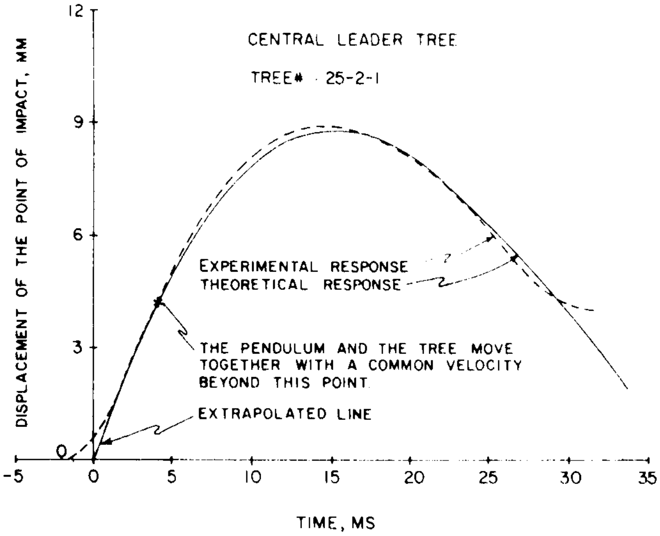

FIG. 5 Experimental and theoretical response of a central leader apple tree.

$$\lim_{i\to\infty} v_i = \lim_{i\to\infty} V_i = V = 0.$$

Rearrangement of equation [32] yields

$$\gamma_{i+1} = (V_{i+1} - v_{i+1}) < \left(\frac{e}{1+e}\right)\left(\frac{M_T+m}{M_T}\right)(v_i - v_{i+1}). \quad [33]$$

For small e, V_{i+1} approaches v_{i+1} rapidly. Since only two or three impacts are experimentally observed to occur before the tree and the pendulum move together, it appears that e is very small.

It is interesting to contrast these considerations with the case in which a billet (an unrestrained mass rather than a pendulum) impacts the tree. In this case ω_2 is zero in equation [25] and it follows that equation [33] becomes an equality.

Once the two bodies move together x and y become equal (Fig. 3) and the equation of motion becomes:

$$(M_T + m)\ddot{x} + C_T\dot{x} + (K_T + k)x = 0 \quad [34]$$

with

$$x = x_o \text{ and } \dot{x} = V \text{ when } t = 0.$$

The well-known solution for equation [34] is

$$x = e^{-bt}\left(x_o \cos\omega t + \frac{(V+bx_o)}{\omega}\sin\omega t\right) \quad [35]$$

where,

$b = \xi\omega_n$
ξ = damping ratio
ω = damped natural frequency
ω_n = undamped natural frequnecy

$$\omega = \omega_n(1-\xi^2). \quad [36]$$

From equation [35] equating dx/dt = 0 for maximum displacement, we get,

$$\tan\omega\tau = V_o\omega/(bV_o + x_o\omega_n^2) \quad [37]$$

where,
τ = time for maximum displacement

Substituting equation [37] in equation [35], we obtain,

$$x_{max} = e^{-b\tau}(V_o^2 + 2bV_o x_o + x_o^2\omega_n^2)^{1/2}/\omega_n.$$

Therefore,

$$\omega_n^2 = \frac{V_o^2 + 2bV_o x_o}{\beta^2 - x_o^2} \quad [38]$$

where

$$\beta = x_{max}e^{b\tau}.$$

For $t << 1$ an order t^2 approximation of equation [35] gives

$$x = e^{-bt}(x_o\cos\omega t + (V+bx_o)/\omega\sin\omega t)$$
$$= (1 - bt + \frac{b^2t^2}{2} + \ldots)[x_o(1-t^2/2 + \ldots) + \frac{(V+bx_o)}{\omega}(\omega t - \ldots)].$$

Using equation [36] and simplifying, we get,

$$x \simeq x_o + Vt - [2bV + x_o\omega_n^2]t^2/2. \quad [39]$$

DETERMINATION OF PHYSICAL PROPERTIES

Using the above theoretical derivation, it is possible to interpret the experimental results to obtain such information as the dynamic mass, the damping ratio, the equivalent spring constant and the Young's modulus of elasticity of the tree material. For the purpose of illustration, data for two tree types, (i) the open center tree and (ii) the central leader tree, were taken from the tree trunk impact fruit harvesting work of Pellerin et al. (1978b). The experimental data from their work are plotted as dashed lines on Figs. 4 and 5. The central leader tree structure refers to a common tree architecture such as the one in Fig. 1. The open center tree structure refers to a modified Y-shaped tree configuration. Pellerin et al. (1978b) have described the experimental procedure used to collect such data. Using a magnetic tape recorder they recorded such information as (i) the angle of the pendulum at release (ii) the displacement of the tree with time upon impact and (iii) whether or not the pendulum was in contact with the tree at each instant of time. If the third piece of information is not present, it is possible to estimate the time at which the pendulum and the tree move together from the displacement vs. time graph. It turns out that each impact corresponds to a sudden increase in the slope of the displacement vs. time plot. Therefore, the last increase in the slope corresponds to the approximate time at which the bodies move together. The experimental results show that this occurs within five milliseconds after the start of the impact process. Using the above information, the physical parameters can be estimated as follows:

1 Obtain x_o and τ from the displacement vs. time graph. τ is noted from the point when the two bodies move together.

2 Digitize the signal for a duration less or equal to ten milliseconds from the point when the two bodies move together.

3 Fit the best fitting polynomial for the data in step (2) using a curvilinear regression analysis (Snedecor and Cochran, 1967).

4 Let the regression yield

$$x = a_o + a_1 t + a_2 t^2 + \ldots \quad [40]$$

5 Equating the right hand sides of equations [39] and [40] we get,

$$x_o = a_o$$

$$V = a_1$$

$$(2bV + x_o \omega_n^2)/2 = -a_2 \quad [41]$$

6 Using equations [38] and [41] the two unknowns, b and ω_n can be determined.

However, step (6) involves solving transcendental equations. Considerable simplification will be shown to occur if the bodies move together within a very short time after the start of the impact process. This situation is thought to occur in most of the problems in which the coefficient of restitution, e, is small. In particular, experimental results suggest that the bodies do move together within a short duration after the impact process for impact-harvesting of fruit trees. In such cases one may extrapolate the displacement vs. time graph linearly from the point when the two bodies move together to the intersection with the time axis; and use this new point as the origin (0,0) for the step (2) above. This extrapolation makes x_o equal to zero and yields:

$$V = a_1$$

$$b\,V = -a_2$$

or

$$b = -a_2/a_1 \quad [42]$$

Using this value of b in equation [38], we obtain

$$\omega_n = V_o/\beta \quad [43]$$

The above procedure resulted in the following illustrative results.

Open Center Tree (Fig. 4)

From the regression:

a_o = -2×10^{-3}, mm
a_1 = V = 1.286 m/s
a_2 = -35.998 m/s^2
x_{max} = 10.375 mm
τ = 16 ms

Therefore,

damping coefficient, b = 28 rad/s
natural frequency, ω_n = 79.2 rad/s
damping ratio, ξ = 0.35

Central Leader Tree (Fig. 5)

From regression:

a_o = -6.7×10^{-4}, mm
a_1 = V = 1.167 m/s
a_2 = -34.817 m/s^2
x_{max} = 8.925 mm
τ = 14.67 ms

Therefore,

damping coefficient, b = 30 rad/s
natural frequency, ω_n = 84.4 rad/s
damping ratio, ξ = 0.36

To estimate the dynamic mass, spring stiffness and Young's modulus of elasticity we proceed as follows:

From equation [15], [20], and [22], we get,

$$M_T V_i + m v_i - (M_T V_{i+1} + m v_{i+1}) = M_T(\omega_1^2 A_x^i + 2\xi\omega_1 \Delta x^i) + m\,\omega_2^2 A_y^i.$$

Summing over all impacts

$$\Sigma[M_T V_i + m v_i - (M_T V_{i+1} + m v_{i+1})] = \Sigma M_T(\omega_1^2 A_x^i + 2\xi\omega_1 \Delta x^i) + \Sigma m\omega_2^2 A_y^i$$

or

$$M_T V_1 + m v_1 - (M_T + m)V = M_T \omega_1^2 A_x + 2 M_T \xi\omega_1 x_o + m\,\omega_2^2 A_y$$

where,

$$A_x = \Sigma A_x^i$$

$$A_y = \Sigma A_y^i$$

$$x_o = \Sigma \Delta x^i.$$

But from equation [20], we have,

$$M_T V_1 + m v_1 = M_T U_1 + m u_1 = m u_1 \quad (\text{as } U_1 = 0)$$

Therefore,

$$m u_1 - (M_T + m)V = M_T \omega_1^2 A_x + m\omega_2^2 A_y + 2 M_T \xi\,\omega_1 X_o$$

or

$$M_T = m(u_1 - V - \omega_2^2 A_y)/(V + \omega_1^2 A_x - 2\xi\omega_1 x_o). \quad [44]$$

A_x and x_o can be obtained from the displacement vs. time graph for the tree. A_y can be obtained from a similar graph for the pendulum. A_x and A_y may also be approximated as

$$A_x \simeq A_y \simeq \frac{1}{2}(x_o T_o)^2 \quad [45]$$

where,

T_o = duration of the impact process, s.

The error due to the approximation in equation [45] is expected to be small if e << 1 as is the case for the tree

TABLE 1. DYNAMIC PARAMETERS OF AN OPEN CENTER AND A CENTRAL LEADER TREE

Description	Open center tree	Central leader tree
Angle of the pendulum at release, θ_r	33.76°	36.39°
Theoretical velocity of the pendulum at impact, u_1	2.235 m/s	2.404 m/s
Actual velocity of the pendulum and the tree when they begin to move together, V	1.286 m/s	1.167 m/s
Actual displacement when the pendulum and the tree begin to move together, x_o	2.4 mm	4.3 mm
Duration of impact process, T_o	4 ms	6 ms
Area A_x and A_y	4.8×10^{-3} mm-s	12.9×10^{-3} mm-s
Dynamic mass of the tree, M_T	87 kg	110 kg
Equivalent spring stiffness of the tree, K_T	1.2×10^6 N/m	1.5×10^6 N/m
Tree trunk diameter, d	142 mm	126 mm
Clamp height, a	354 mm	315 mm
Young's modulus of the tree trunk, E	8.9×10^9 N/m²	12.6×10^9 N/m²

The actual displacements were obtained from the experimental data of Pellerin et al. (1978b), who estimated the velocities by measuring the slope of the displacement - time curves.

impact problem.

We have,

$$\frac{K_T + k}{M_T + m} = \omega_1^2$$

or

$$K_T = \omega_1^2 (M_T + m) - k \quad [46]$$

From equation [9], $K_T = 3EI/a^3$

Therefore,

$$E = \frac{K_T a^3}{3I} \quad [47]$$

The above parameters were estimated for the open center and the central leader trees. They are tabulated in Table 1. The theoretical velocities listed in Table 1 for the pendulum were obtained from a consideration of the conservation of mechanical energy for the pendulum. Neglecting the friction at the point of suspension and assuming that the impact occurs at the center of percussion of the pendulum, the conservation of mechanical energy computations yield the following expression for u_I

$$u_1 = 2\sqrt{g\,\ell_e}\sin(\theta_r/2) \quad [48]$$

where,

θ_r = angle at release of the pendulum, deg.

ℓ_e = equivalent length of a simple pendulum, m = 1.51 m

g = acceleration due to gravity, m/s² = 9.81 m/s²

In calculating the parameters listed in Table 1, the following values were used for the parameters for the pendulum.

The mass of the pendulum: 132.4 kg

The equivalent spring constant, mg/l_e = 860 N/m

The natural frequency, $\omega_2 = \sqrt{g/l_e}$ = 2.55 rad/s

Note that if a heavy, spring loaded mass is used as an impacting device, then in the above analysis $\omega_2 = \sqrt{g/L_e}$ is substituted as zero. The theoretical velocity, u_1 is obtained by an appropriate energy consideration for the impacting device.

DISCUSSION

Figs. 4 and 5 indicate that a one degree of freedom model closely represents the dynamic behavior of the tree at the time of impact and for a short time immediately after the impact. Later differences are seen between the theoretical and experimental curves. These differences are mainly due to the removal of the fruit. The fact that the open-center tree shows a greater deviation than does the central-leader tree further strengthens the above argument. Removal of the fruit means that a relatively higher percent of total mass is lost for an open-center tree than for a central-leader tree. Furthermore, factors such as the departure from a one degree of freedom model, the visco-elastic nature of the biological tissues and the external non-linear damping due to the leaves and twigs could have also affected the response.

Our theory predicts an infinite number of impacts when a pendulum or a billet impacts a tree or an elastic member. The process converges in finite time, after which the bodies move together for a while. If e is small, the process converges very rapidly. Under such circumstances only two or three distinctly visible impacts may be observed in a real system. The rest of the impacts are difficult to observe. This appears to be the case for the impact-harvesting of fruit trees.

The fact that the two bodies move together for a short duration following the impact process has been used to predict the physical properties of the tree such as its dynamic mass, damping coefficient, spring stiffness, Young's modulus and natural frequency.

These results should be useful in the design of impact type tree fruit harvesting machinery and in the interpretation of experimental results. Pellerin et al. (1979, unpublished report), used this one degree of freedom model with physical parameters estimated in the illustrative examples of this study in the design of a double-impact tree trunk impact fruit harvester. They neglected damping for simplicity. The results of their field test shows that their objective of timing the duration between the two impacts met with reasonable accuracy.

Notes on Energy Transfer

Pellerin et al. (1978b) found that the difference between the impact and rebound energy of the pendulum did not correlate well with the fruit removal. Scura (1976) also found that the loss of the pendulum energy due to an impact onto a soil column was not a good indication of the actual energy transmitted to the soil column. *In this analysis we assume that the energy transmitted due to an impact is less than the difference in the energy level of the pendulum before and after im-*

pact by an amount equal to the loss of mechanical energy due to the impact process.

We shall elaborate on this point. The loss of mechanical energy due to the i^{th} impact, δ_i is

$$\delta_i = \frac{1}{2}\, m(u_i^2 - v_i^2) + \frac{1}{2}\, M_T\,(U_i - V_i)^2 . \qquad [49]$$

From equation [20]

$$mu_i + M_T\,U_i = mu_i + M_T V_i$$

or

$$m(u_i - v_i) = -M_T(U_i - V_i) . \qquad [50]$$

From equations [49] and [50], we get

$$\delta_i = \frac{m}{2}\,(u_i - v_i)\left\{(u_i - U_i) + (v_i - V_i)\right\} . \qquad [51]$$

But from equation [21]

$$v_i - V_i = -e(u_i - U_i). \qquad [52]$$

Therefore, from equations [51] and [52]

$$\delta_i = \frac{m}{2}\,(1-e)(u_i - v_i)\,(u_i - U_i). \qquad [53]$$

Eliminating V_i from equations [50] and [53], we obtain

$$v_i = \left(\frac{m - M_T e}{M_T + m}\right) u_i + \left(\frac{M_T(1+e)}{M_T + m}\right) U_i . \qquad [54]$$

Substituting for v_i in equation [53], we get

$$\delta_i = \frac{M_T m}{2(M_T + m)}\,(1-e^2)(u_i - U_i)^2 . \qquad [55]$$

Summing over all the impacts, the total mechanical energy lost, Δ, is given by

$$\Delta = \Sigma\,\delta_i = \frac{M_T m}{2(M_T + m)}\,(1-e^2)\Sigma(u_i - U_i)^2 . \qquad [56]$$

Note that for a perfectly elastic impact $e = 1$, the energy lost is zero. On the other hand, for a purely inelastic impact $e = 0$, and the total mechanical energy lost, Δ, is

$$\Delta = \frac{M_T m}{2(M_T + m)}\, u_1^{\,2} . \qquad [57]$$

When e is zero, there is only one impact (equation [52]). So for a soil column or tree impact problem for which e is expected to be small, the mechanical energy lost due to impact is approximated by equation [57]. If θ_b is the rebound angle of the pendulum (Fig. 1), then the total energy transferred to the tree due to an impact, E_T, is:

$$E_T = mg\,\ell_e(1-\cos\theta_r) - mg\,\ell_e(1-\cos\theta_b) - \frac{M_T m}{2(M_T+m)}\, u_1^{\,2}$$

$$= mg\,\ell_e(\cos\theta_b - \cos\theta_r) - \frac{M_T m}{2(M_T+m)}\, u_1^{\,2} \qquad [58]$$

Therefore, we suspect that fruit removal and fruit damage may be better correlated to the expression given in equation [58] than simply the difference in potential energy before and after impact. To illustrate this point let us consider our previous example. For the open-center tree with a rebound angle of 4 deg the energy lost due to the impact is about 40 percent of the difference in the mechanical energy before and after the impact. For the central leader tree with a rebound angle of 6 deg this difference is about 47 percent. Thus, the energy lost due to the impact may be a significant portion of the difference in mechanical energy before and after the impact. Note that equation [57] also represents the mechanical energy lost when a spring loaded mass is used to impact the tree.

CONCLUSIONS

1 Our analysis shows that a one degree of freedom model can describe the motion of a given point on a tree trunk or an elastic member when it is impacted by a moving mass.

2 Our model predicts the existence of an infinite number of impacts when the tree or an elastic member is struck by a heavy pendulum or a billet (heavy mass). However, following these impacts the bodies move together with a common velocity for a short duration.

3 We have provided a simple, approximate way of determining the dynamic parameters of the tree.

4 We have also provided an estimation of the energy transferred to the tree due to an impact, equation [58]. We believe that this equation will correlate better with fruit removal and fruit damage than the difference in the energy level of the impacting device before and after the impact. The error due to neglecting the loss in the mechanical energy may be as high as 40 to 50 percent of the energy input. This error raises doubts about the conclusions of the previous studies in which this technique has been used.

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