

Multibody Dynamics A

wb1310

Arend L. Schwab
Laboratory for Engineering Mechanics
Delft University of Technology

Spring 2009
2nd lecture

Contents

Lecture	Topic	Assignment
1th	Introduction, Team-up.	1–Pendulum
2nd	Newton-Euler eqns of motion for a 3D rigid body.	2–Wheel
3rd	Modelling of Mechanical Systems.	3–Crane
4th	Orientation of a Rigid Body in Space.	4–Governer
5th	Coupled Differential and Algebraic equations.	5–Tractor
6th	Overview.	5–Tractor

Newton-Euler equations of motion for a rigid body in space:

$$\text{Newton: } \sum \mathbf{f}_c = m_c \ddot{\mathbf{x}}_c.$$

$$\text{Euler: } \sum \mathbf{M}_c = \mathbf{I}_c \dot{\boldsymbol{\omega}}_c + \boldsymbol{\omega}_c \times (\mathbf{I}_c \boldsymbol{\omega}_c).$$

with, at the centre of mass c , the total mass m_c , and the mass moments of inertia tensor \mathbf{I}_c :

$$\mathbf{I}_c = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

where $I_{xx} = \int (y^2 + z^2) dm$ and $I_{xy} = - \int (xy) dm$, other terms are obtained by cyclic permutation of x, y, z .

From these we can calculate for a given set of forces $\sum \mathbf{f}_c$ and moments $\sum \mathbf{M}_c$ acting at the centre of mass of the rigid body, in a given state, the accelerations of the centre of mass $\ddot{\mathbf{x}}_c$ and the angular acceleration $\dot{\boldsymbol{\omega}}_c$ of the rigid body.

Note that in the Euler equations the components of the inertia tensor \mathbf{I}_c change due to the rotation of the body!

However, in a body fixed coordinate system, (x', y', z') , the inertia tensor of a rigid body, \mathbf{I}'_c , is constant.

Transformation of the Euler equations of motion from the space fixed coordinate system (x, y, z) into the body fixed coordinate system (x', y', z') result in:

$$\sum \mathbf{M}'_c = \mathbf{I}'_c \dot{\boldsymbol{\omega}}'_c + \boldsymbol{\omega}'_c \times (\mathbf{I}'_c \boldsymbol{\omega}'_c)$$

with constant \mathbf{I}'_c !

Newton-Euler equations for a rigid body derived by means of linear and angular momentum balance.

Linear Momentum: $\mathbf{p}_c = m_c \mathbf{v}_c$.

Angular Momentum: $\mathbf{H}_c = \mathbf{I}_c \boldsymbol{\omega}_c$

Taking the rate of change, we get:

Linear Momentum Balance: $\sum \mathbf{f}_c = \dot{\mathbf{p}}_c$.

Angular Momentum Balance: $\sum \mathbf{M}_c = \dot{\mathbf{H}}_c$.

Now with the Rotating Axes Theorem:

$$\frac{d}{dt}(\mathbf{X})_{fixed} = \frac{d}{dt}(\mathbf{X})_{moving} + \boldsymbol{\Omega}_{moving} \times (\mathbf{X}),$$

where \mathbf{X} is some vector quantity and $\boldsymbol{\Omega}_{moving}$ is the angular velocity of the moving coordinate system with respect to the fixed coordinate system.

We get, again, the Newton-Euler equations as:

$$\sum \mathbf{f}_c = m_c \dot{\mathbf{v}}_c$$

$$\sum \mathbf{M}_c = \mathbf{I}_c \dot{\boldsymbol{\omega}}_c + \boldsymbol{\omega}_c \times (\mathbf{I}_c \boldsymbol{\omega}_c).$$

Assignment 2

In order to examine the steering forces and moments of a bicycle in motion, the front wheel of a bike is put to a further test. The model of this front wheel consists of a thin hoop with a mass of $m = 1.8$ [kg] and a diameter of $d = 28''$, which can rotate around its own axle, ϕ . It is assumed that the mass is concentrated along the perimeter of the wheel. Perpendicular to this axis of rotation, a second hinge has been attached in order to be able to rotate the ϕ -axle, the so-called steering, around an angle ψ .

1. Make an estimate, by means of the Euler equation of motion for a rigid body, $\mathbf{M} = \mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega})$, of the size and direction of the moment \mathbf{M}_1 that is exerted on the wheel in the first hinge at a constant riding speed of $v = 20$ [km/hour] and a constant steering angular velocity of $\dot{\psi} = 30$ [$^\circ$ /sec].