

Multibody Dynamics A

wb1310

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Spring 2009
5th lecture

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Coupled Differential and Algebraic Equations (DAE's)

For the dynamic analysis of systems of interconnected rigid bodies, the so-called multibody systems, we have the Newton-Euler equations of motion for the rigid bodies together with the algebraic equations expressing the interconnections or constraints. Together these result in a set of coupled differential and algebraic equations (DAE's) from which we can solve the motion.

The principle of virtual power tells us something about the 'equilibrium' of the system.

Proposition: The system is in equilibrium if the virtual power of the applied forces is zero for all virtual velocities which fulfill the constraints.

d'Alembert's Tric: Transform a dynamics problem into a statics problem by adding the inertia forces to the applied forces.

The virtual power of the applied forces together with the inertia forces is now

$$(f_i - M_{ij}\ddot{x}_j)\delta\dot{x}_i = 0,$$

with $i, j = 1..n$, and $n = 6*b$ where b is number of bodies.

In general the constraints can be written as:

$$C_k(x_i, t) = 0,$$

with $k = 1..m$, where m is the total number of constraints, e.g. for s hinges $m = 5 * s$.

Differentiation with respect to time results in m constraints on the velocities \dot{x}_i :

$$\dot{C}_k = \frac{\partial C_k}{\partial x_i} \dot{x}_i + \frac{\partial C_k}{\partial t} = 0.$$

The virtual velocities have to fulfill these velocity constraints (virtual = time stands still):

$$\delta\dot{C}_k = \frac{\partial C_k}{\partial x_i} \delta\dot{x}_i = 0.$$

These constraints on the virtual velocities can be added to the virtual power by means of the (unknown) Lagrange multipliers λ_k , as many as we have constraints $\delta\dot{C}_k = 0$:

$$(f_i - M_{ij}\ddot{x}_j)\delta\dot{x}_i = \lambda_k \delta\dot{C}_k,$$

Substitution of the expressions for the virtual velocities results in:

$$(f_i - M_{ij}\ddot{x}_j)\delta\dot{x}_i = \left(\lambda_k \frac{\partial C_k}{\partial x_i}\right)\delta\dot{x}_i,$$

This must hold for arbitrary virtual velocities $\delta \dot{x}_i$, therefore the scalar virtual power equation results in i differential equations:

$$M_{ij} \ddot{x}_j + \frac{\partial C_k}{\partial x_i} \lambda_k = f_i,$$

These are the n differential equations of motion with $n + m$ unknowns, namely: \ddot{x}_i and λ_k .

Next add the m algebraic equations for the constraints. But since the unknowns are the accelerations we will differentiate these constraints twice with respect to time :

$$C_{k,i} \ddot{x}_i + C_{k,ij} \dot{x}_i \dot{x}_j + 2 \frac{\partial C_{k,i}}{\partial t} \dot{x}_i + \frac{\partial^2 C_k}{\partial t^2} = 0,$$

where we use the comma notation to express partial derivatives, e.g. $C_{k,i} = \frac{\partial C_k}{\partial x_i}$.

Finally we have the complete set of coupled Differential and Algebraic Equations (DAE's):

$$M_{ij}\ddot{x}_j + \frac{\partial C_k}{\partial x_i}\lambda_k = f_i,$$

$$C_{k,i}\ddot{x}_i = -ddC_k.$$

with the 'known' acceleration terms

$$ddC_k = C_{k,ij}\dot{x}_i\dot{x}_j + 2\frac{\partial C_{k,i}}{\partial t}\dot{x}_i + \frac{\partial^2 C_k}{\partial t^2}.$$

Or in mixed Matrix-Vector-Index notation:

$$\begin{bmatrix} M_{ij} & C_{k,i} \\ C_{k,j} & 0_{kk} \end{bmatrix} \begin{bmatrix} \ddot{x}_j \\ \lambda_k \end{bmatrix} = \begin{bmatrix} f_i \\ -ddC_k \end{bmatrix}.$$

The motion of the system can now be found by numerical integration of the time derivatives of the *state* (state equations). Where the *state* of the system is given by the moment in time t , the positions of the centres of mass of the rigid bodies $(x, y, z)_i$ ($i = 1..b$, where b is the number of rigid bodies), the orientation of the rigid bodies expressed in f.i. the Euler angles $(\varphi, \chi, \psi)_i$, the velocities of the centres of mass $(\dot{x}, \dot{y}, \dot{z})_i$, and the angular velocities of the rigid bodies $(\omega'_x, \omega'_y, \omega'_z)_i$ expressed in the body fixed frame $x'y'z'$.

These state equations are:

$$\begin{aligned}
 d(x, y, z)_i/dt &= (\dot{x}, \dot{y}, \dot{z})_i \\
 d(\varphi, \chi, \psi)_i/dt &= A((\varphi, \chi, \psi)_i)(\omega'_x, \omega'_y, \omega'_z)_i \\
 d(\dot{x}, \dot{y}, \dot{z})_i/dt &= (\ddot{x}, \ddot{y}, \ddot{z})_i \quad \text{from the DAE's} \\
 d(\omega'_x, \omega'_y, \omega'_z)_i/dt &= (\dot{\omega}'_x, \dot{\omega}'_y, \dot{\omega}'_z)_i \quad \text{from the DAE's}
 \end{aligned}$$

with, for (313) Euler angles, the A matrix as

$$A((\varphi, \chi, \psi)_i) = \frac{1}{s_{\chi_i}} \begin{pmatrix} s_{\psi} & c_{\psi} & 0 \\ s_{\chi} c_{\psi} & -s_{\chi} s_{\psi} & 0 \\ -c_{\chi} s_{\psi} & -c_{\chi} c_{\psi} & s_{\chi} \end{pmatrix}_i .$$

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