

wb1413

Multibody Dynamics B

Spring Term 2009, Thu 15:45-17:30, Mechanical Engineering, room C, 4 ECTS credits.

Homework assignment 7

Determine the motion of the double pendulum from assignment 1 by numerical integration of the equations of motion expressed in independent generalized coordinates as derived in assignment 5. The initial conditions are both bars vertically up at zero speed. We assume a gravitational field operating in the *horizontal* direction with a field strength of $g = 9.81$ N/kg. We want to determine the angle, in radians, of both bars with respect to the horizontal axis after 3.5 seconds with a maximal absolute error of 10^{-5} rad.

a. Determine the accordingly maximum step size for the following numerical integration methods:

- (1) Euler.
- (2) Heun.
- (3) Runge-Kutta 3rd order.
- (4) Classical Runge-Kutta 4th order.

Use an error estimate method based on the method-inherent truncation error and the round-off error due to the finite precision as explained in the course. Plot for each angle in one figure the \log_{10} (estimated error) versus the \log_{10} (step size) for all four methods. In some cases it may not be feasible to find such a maximum step size. Either the method used is unstable or the step size becomes so small that it is impractical to reach the end point at $t=3.5$ seconds. If so, please do not spend too much time on that and state your case clearly!

b. Finally use the three ODE solvers `ode23`, `ode45`, and `ode113` from Matlab. Set the error tolerance `RelTol` and `AbsTol` such that you get the same final accuracy (global error!) as above and integrate the equations of motion for 3.5 seconds.

- (1) Compare the angles of both bars at $t = 3.5$ sec with the results from above.
- (2) Determine the average step size and the total number of function evaluations (calls to the differential equation $f(t,y)$) as used in the three methods. Do these agree with your previous results?

Please tabulate all your results (with enough digits [for instance 15] to show convergence) and discuss.

Bonus Question: What happens when you start a simulation with initial conditions very close to those from above? Do the solutions stay close together? How do we call such a behaviour?

References

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- [5] John H. Hubbard and Beverly H. West. *Differential Equations: A Dynamical System Approach, Part 1, Ordinary Differential Equations*. Number 5 in Texts in Applied Mathematics. Springer, New York, 1991.
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